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Characteristics of the modulus of passive resistance of soil

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CHARACTERISTICS OF THE MODULUS OF PASSIVE RESISTANCE OF SOIL

by

Reynold King Watkins

**A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
DOCTOR OF PHILOSOPHY**

Major Subject: Soil Engineering

Approved:

Signature was redacted for privacy.

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Iowa State College

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NOTATION

The notation in this report as listed below generally agrees with notation commonly used in present day literature on soil mechanics. A major exception is the notation of Appendix D which is taken from Timoshenko and Goodier (25). That notation is defined in Appendix D and is not listed here.

- a = minimum (vertical) semi-diameter of deformed pipe if cross section is assumed elliptical.
- B = breadth of rectangular bearing surface.
- b = maximum (horizontal) semi-diameter of deformed pipe if cross section is assumed elliptical.
- C = coefficient used in the Marston pipe load formula.
- C_1 = slope of Spangler's plots of C as a function of $\frac{H}{D}$ for incomplete ditch condition.
- C_1 = constant for a given set of soil characteristics which relates the deformation of the pipe with the displacement of the soil.
- C_2 = intercept of Spangler's plots of C as a function of $\frac{H}{D}$ for incomplete ditch condition.
- c = cohesion of soil.
- D = mean diameter of pipe.
- E = Modulus of Elasticity.

- e = modulus of passive resistance of soil.
 e_r = modulus of soil reaction.
 e_v = void ratio of soil.
 f, f^1 = symbol for functional relationship.
 H = height of earth fill above the top of the pipe.
 H_e = height of fill from top of pipe to plane of equal settlement.
 H_0 = height of fill supported by interlocking of the soil grains.
 h = horizontal soil pressure against flexible pipe culvert at horizontal diameter.
 I = moment of inertia of the cross section of the pipe wall per unit length of pipe.
 K = coefficient in the Iowa Formula which depends on the pipe bedding angle.
 k = eccentricity of ellipse.
 k_h, k_s = Terzaghi's coefficient of subgrade reaction.
 L = length or length dimension.
 l = length of pipe.
 m = constant of proportionality between k_h or e and H in sand according to Terzaghi's theory.
 m as a subscript refers to a model.
 m' = constant of proportionality between h and x according to Terzaghi's theory.
 n = length scale factor.

- P = net innertube pressure.
- P_o = P-intercept of load-deflection diagram.
- p = projection ratio.
- P_v = vertical intergranular soil pressure.
- P_x = horizontal intergranular soil pressure.
- q = increase in vertical load per unit area on a horizontal slab or loading plate.
- r = mean radius of pipe.
- r_{sd} = settlement ratio.
- S = slope of load-deflection diagram.
- s = circumference of pipe.
- T = transmission ratio of a given soil in a given model cell = ratio of p_v to P at the level of the top of the pipe.
- t = thickness of the pipe wall.
- W_c = vertical soil load per unit length of pipe at the level of the top of the pipe.
- w = water content of soil.
- X = horizontal coordinate to point of intersection of coaxial ellipse and circle with equal circumferences.
- X-boundary = plane perpendicular to the x-axis which is a boundary of relative soil displacement due to deformation of the pipe.
- x = horizontal axis or coordinate in the plane of the pipe cross section.

- Y = vertical coordinate to point of intersection of coaxial ellipse and circle of equal circumferences.
- Y-boundary = plane perpendicular to the y-axis which is a boundary of relative soil displacement due to deformation of the pipe.
- y = vertical axis or coordinate in the plane of the pipe cross section.
- Z-boundary = plane perpendicular to the z-axis which is a boundary of relative soil displacement due to deformation of the pipe.
- z = axis or coordinate in the direction of the pipe axis.
- α = bedding angle of the flexible pipe.
- β = $\tan^{-1} \frac{y}{x}$.
- γ = unit weight of soil.
- Δ = $b - a$.
- Δx = increase in horizontal diameter of flexible pipe from initial circular shape to deformed shape.
- Δy = decrease in vertical diameter of flexible pipe from initial circular shape to deformed shape.
- θ = half of the angle of passive bearing surface on flexible pipe under an earth fill.
- λ = any pertinent length in the soil.
- π_s = pi-term.
- ϕ = internal friction angle of soil.
- Ω = compactive effort in the soil.

I. INTRODUCTION

The increased number and size of earth fills in modern highway construction is apparent to all who travel on the Nation's highways. The vast expansion of the highway program alone accounts for many new earth fills, but in addition the percentage of new highway mileage on earth fills is increasing. This is easily accounted for since new highways are generally designed with curves which are fewer in number and greater in radius--both horizontal and vertical--so that sight distance might be improved for high speed motor vehicles. The amount of fill is an inverse function of the number and degree of the curves. Railroads, to a lesser degree, are also constructing many new earth fills.

Furthermore there is a decided trend toward the design of earth fills for conditions under which bridges and trestles would have been used in the past. Indeed many old bridges and trestles are being replaced by earth fills. A good example is Southern Pacific Railroad's 49 million dollar rock fill which will replace the old Lucin Cutoff trestle in the Great Salt Lake. This trend toward earth fills is not limited to highway or railway construction. Earth fills are now being used in place of concrete for many new dams. They are replacing sheet piling in many new cut-off walls, levees,

cofferdams, etc. Even structures such as buildings, air strips, piers, etc., are being placed on earth fills which would not have been economical in the past.

The expanding use of earth fills is no accident. The development of modern high-speed, high-powered earth moving equipment is largely responsible. Unit costs of hauling earth have steadily decreased as the efficiency of the equipment has increased. On the other hand, the cost of steel and steel construction, and the cost of labor in concrete construction has increased markedly.

With the increased use of earth fill, there is a corresponding step-up in the development of drainage systems involving culverts and drain tile and pipe. Development has been the greatest in the case of preconstructed pipe, both rigid and flexible. As is typical of design methods in all rapidly expanding construction systems, rules of thumb have emerged from trial and error installations, and empirical methods have developed from the rules of thumb. Now rational theory is supplanting the empirical methods which have been found inadequate.

In 1913 Dean Anson Marston of Iowa State College published a theory for calculating loads on conduits embedded in soil. The theory is now generally accepted. The design of rigid pipe follows immediately from the prediction of load by the Marston theory. Flexible pipe poses a different

problem, however, since much of the pipe's strength is developed by the surrounding soil which supports the pipe laterally as the pipe deforms. Consequently a theory has been proposed by M. G. Spangler, Research Professor of Civil Engineering of Iowa State College, for designing flexible pipe by predicting pipe deflection. His theory has not yet found general application because of insufficient knowledge of the relationship between lateral soil pressure and lateral deflection of the pipe. This relationship has been written as a ratio by Spangler and is called the modulus of passive resistance. If this modulus could be evaluated, there is little doubt that the rational design of flexible pipe would supplant existing empirical methods.

This dissertation is the report of a study designed to further the understanding of the modulus of passive resistance.

II. MODULUS OF PASSIVE RESISTANCE

A. Theory

During the construction of a soil fill over a flexible pipe culvert, the vertical diameter of the pipe decreases, and the horizontal diameter increases as vertical load on the pipe is increased. The increasing horizontal diameter causes the pipe to bear laterally with increasing force against the adjacent soil. The greater the lateral bearing resistance of the soil, the less will be the deformation of the pipe, and the less will be the chance of failure. It has been demonstrated by experience that failure in flexible pipe culvert may be acceptably defined in terms of excessive deformation (1, p. 70 and 19, p. 340). Maximum lateral support is developed when the horizontal deflection reaches a maximum, but at this point the pipe is in a state of incipient collapse, for any additional vertical load causes reversal of curvature of the top portion of the pipe section. See Figures 1a and 1b. Reversed curvature decreases the horizontal diameter; the benefit of the lateral soil support is lost; and failure results. Thus the maximum vertical load is developed at approximately the point of maximum horizontal deflection.

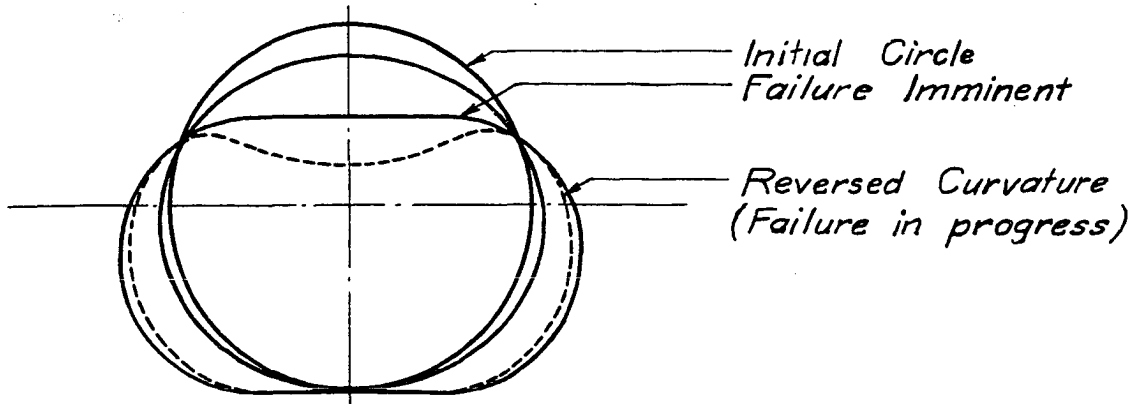


Figure 1a. Stages of deformation of a flexible pipe under a soil fill up to reversal of curvature which is considered to be failure

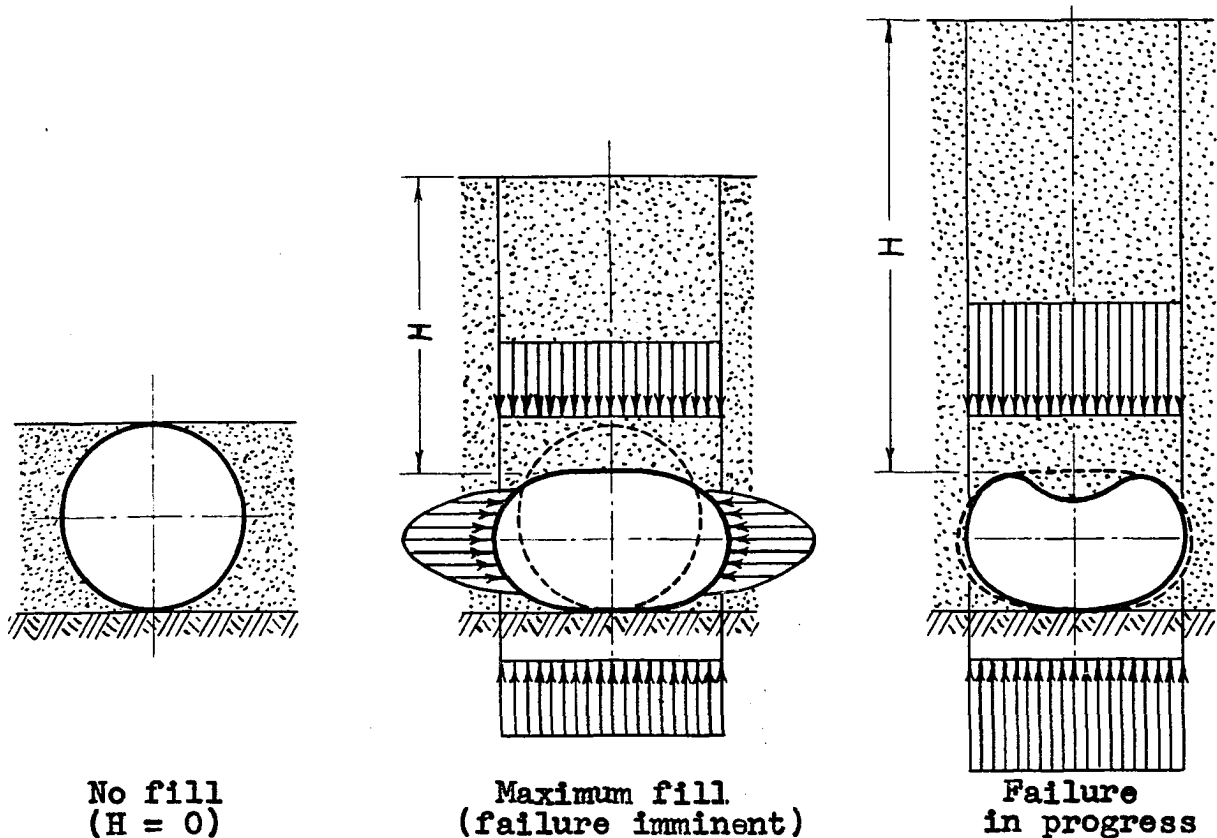


Figure 1b. Progressive stress patterns on flexible pipe into the stage of reversed curvature at which lateral support is lost and failure occurs

Moreover, it has been observed that for a given per cent increase in horizontal deflection the configuration of the pipe cross section is approximately the same for all flexible pipe culverts. Engineers of the Armco Drainage and Metal Products, Inc. (1, p. 70), claim that maximum vertical load causes about a 20 per cent decrease in the vertical diameter of the pipe. From their experience it has generally become customary to define failure conditions in a flexible pipe culvert as 20 per cent decrease in the vertical diameter. As a basis for design, a decrease of 5 per cent in the vertical diameter is used by Armco.

For small deformations of this magnitude (5 per cent), the vertical decrease in diameter is approximately equal to the horizontal increase in diameter. If the pipe remained elliptical in shape during deformation and if the circumference of the culvert remained constant, the horizontal increase in diameter, Δx , would be related to the vertical decrease in diameter, Δy , by the relationship

$$\Delta x = 0.914 \Delta y \quad (18, \text{ p. } 14).$$

For practical design purposes it makes no difference whether the horizontal or vertical deflection is specified (18, p. 29). In this thesis the horizontal deflection is the basis for consideration since it deals more directly with the lateral bearing resistance of the soil.

In order to design flexible pipe culverts according to

the deflection concept, Spangler (18, p. 29) has derived a rational formula for predicting increase in horizontal diameter, Δx , for a pipe embedded in a soil fill. He refers to his formula as the Iowa Formula. His formula follows:

$$\Delta x = \frac{K W_c r^3}{E I + 0.061 (er) r^3} \quad \text{Eq. 1}$$

$$\begin{aligned} \text{where } K = & 0.5 \sin \alpha - 0.082 \sin^2 \alpha + 0.08 \frac{\alpha}{\sin \alpha} \\ & - 0.16 \sin \alpha (\pi - \alpha) - 0.04 \frac{\sin 2\alpha}{\sin \alpha} \\ & + 0.318 \cos \alpha - 0.208. \end{aligned}$$

Note: K is a function of α only, and is assumed constant for any given installation. See Figure 2 for dimensions. In these equations:

Δx = increase in horizontal diameter of the pipe culvert (in.)

W_c = vertical load per unit length of the pipe at the level of the top of the pipe (lb./in.)

r = mean radius of the pipe (in.)

E = modulus of elasticity of the material from which the pipe is constructed (lb./in.²)

I = moment of inertia of the cross section of the pipe wall per unit length along the pipe (in.⁴/in.)

e = modulus of passive resistance (lb./in.²/in.)

α = bedding angle of the pipe (degrees or radians).

The above notation is all familiar to engineers except the

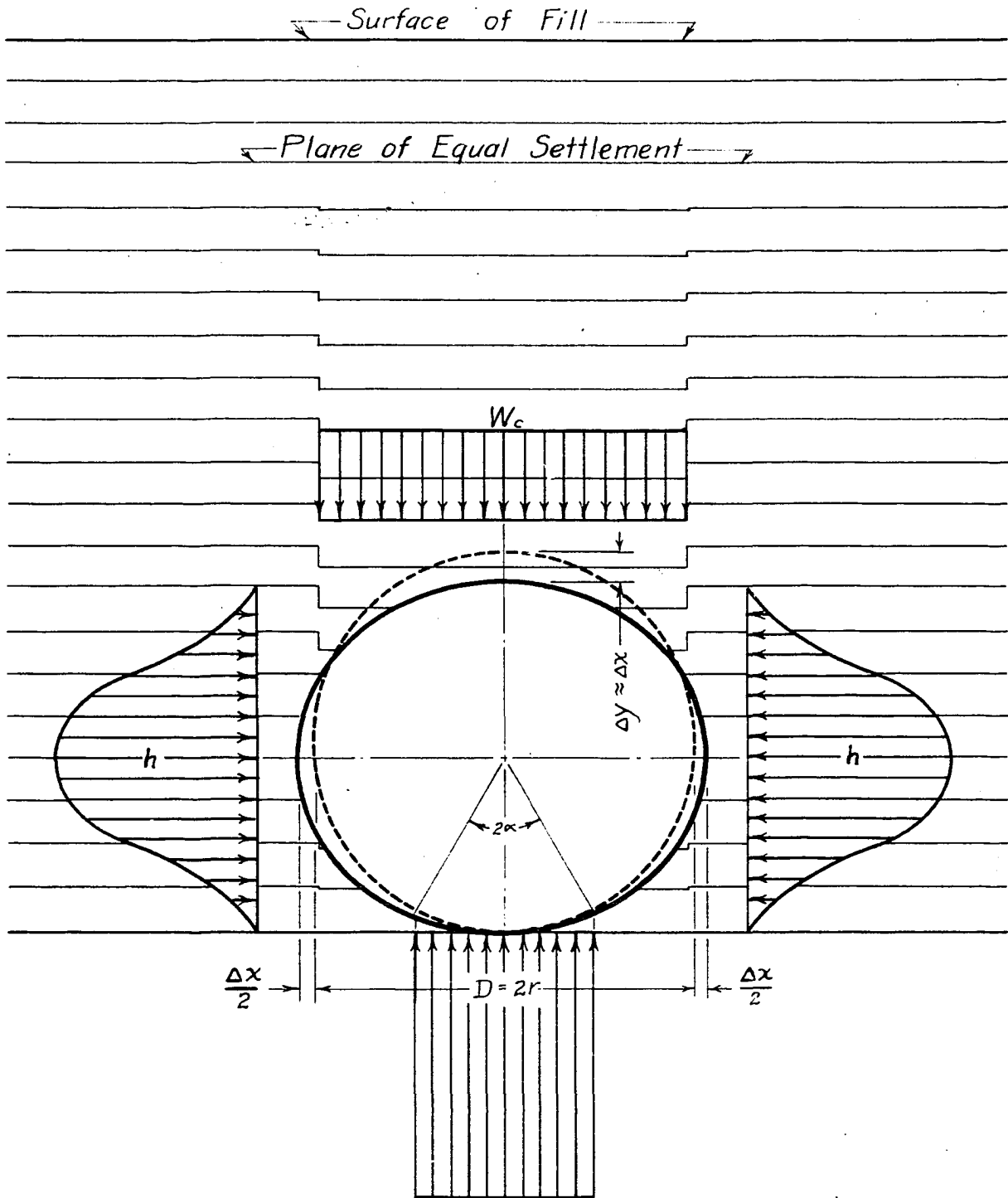


Figure 2. Idealized direct stress pattern on a flexible pipe culvert under an earth fill resulting from idealized soil displacement

quantity, e ; and all quantities can be easily evaluated except e . The quantities r and α can be measured in any given installation. E and I occur only in the form of a stiffness factor, EI , which can be evaluated for any given pipe by tests or by simple principles of mechanics and strength of materials. W_c can be determined by means of the Marston formula (16, pp. 422, 424) for culvert loading. The Marston formula follows:

$$W_c = C \gamma (2r)^2 \quad \text{Eq. 2}$$

where γ = unit weight of the soil fill (usually given in lb./ft.³)

r = mean radius of the pipe (usually in.)

C = coefficient depending on: (a) the ratio of the height of fill to diameter of the pipe, $\frac{H}{D}$, and (b) settlement ratio and projection ratio which define the pipe condition; i.e., incomplete or complete, ditch or projection according to Marston as described by Spangler (16, pp. 409-427).

The major stumbling block to the use of the Iowa Formula appears to be the evaluation of e . This quantity was invented by Spangler (18, p. 28) who referred to it as the modulus of passive resistance. It is a measure of the lateral bearing resistance or support contributed by the adjacent soil as referred to in the opening paragraph. The

modulus of passive resistance is similar to Westergaard's modulus of subgrade reaction (27), Cumming's modulus of foundation (5) and Terzaghi's coefficient of horizontal subgrade reaction (22, 23) in that it is a measure of the rate of change of lateral pressure with respect to lateral displacement. Written mathematically,

$$e = \frac{2 d (h)}{d (\Delta x)} \quad \text{Eq. 3}$$

where h = maximum horizontal soil pressure against the pipe assumed to act at the extremity of the horizontal diameter. See Figure 2.

$\frac{\Delta x}{2}$ = horizontal displacement of the pipe at the point on which h acts.

It is interesting to note that the dimensions of e are FL^{-3} where F represents force and L represents length. This agrees with the similar quantities developed by Westergaard, Cummings and Terzaghi.

As would be expected, Formula 1 shows that the greater the modulus of passive resistance, the less the deflection, Δx , and the less the chance for failure. If e were zero the pipe culvert would fail by deformation under a relatively small vertical load which would cause it to collapse for lack of lateral support. On the other hand, if e were very large, the pipe would support a tremendous load up to the condition at which the pipe wall would fail by crushing or buckling.

Since modulus of passive resistance, e , describes this range of values, it is evidently a most important factor in the economical design of flexible pipes embedded in soil. Unfortunately it is ignored in most present-day design. So little is known about it, that designers customarily resort to over-simplified experience tables with a factor of safety of four or more. For example in the Armco "Handbook of Drainage and Construction Products" (1, p. 105) design tables for flexible steel pipe culvert specify a gage number (thickness of metal) for a given diameter of flexible steel pipe as a function of height of fill. Each gage number listed is calculated by Shafer's empirical formula (Equation 4) from an average of numerous actual pipe installations in which the vertical diameter has decreased by 5 per cent for a given height of fill regardless of the type of fill or the degree of compaction. This 5 per cent deflection represents a safety factor of 4 since it is assumed throughout that 20 per cent change in the vertical diameter is a definition of failure. No attempt is made whatsoever to consider the effect of the modulus of passive resistance on the gage of metal to be specified for a given installation. Such design practices need alteration for, despite the possibility of overdesign (with its related lack of economy), designers still suffer the embarrassment of occasional failures. Moreover such design practices offer no incentive for care

in specifying degree of compaction or selection of fill.

Although the modulus of passive resistance, e , appears to be the key to rational design of flexible pipe culverts, it is not yet understood. Not only are there no reliable quantitative values available for e , but its characteristics have not even been established to the general satisfaction of designers. The following historical background indicates the extent to which conflicting concepts have arisen regarding the characteristics of e .

B. Historical Background

Spangler's Iowa Formula was first published in Iowa Engineering Experiment Station Bulletin 153 in 1941. In connection with the derivation, Spangler (17, pp. 31-58) reported the results of three series of experiments designed to test the applicability of the formula. In general the experiments showed that the formula may be used successfully for predicting the deflection of flexible pipe culverts if an appropriate value can be found for the modulus of passive resistance, e , of the soil. Spangler also observed some of the characteristics of e in his early experiments. Since these observations form a starting point for this investigation, they are summarized below:

1. Qualitatively, the greater the density of the soil

adjacent to the pipe, the greater the modulus of passive resistance. It was found that even with compaction much below standard Proctor density, the value for e was twice as great as with uncompacted fill (18, p. 65).

2. The modulus of passive resistance, e , appears to be independent of the height of fill, both for uncompacted and compacted fill (18, p. 24 and 19, p. 343).

3. The modulus of passive resistance, e , appears to be a function of the properties of the soil only (18, p. 5), that is, e is a constant for any given set of soil conditions. This statement is further implied by Spangler's use of e in his derivation of the Iowa Formula (18, p. 28).

In 1948 a summary of Spangler's work on flexible pipe culverts was included in a paper presented to the American Society of Civil Engineers (19). The paper was entitled "Underground Conduits--An Appraisal of Modern Research." The paper drew discussion from a number of leading engineers who had considerable experience in culvert design. Shafer (15, p. 357) recognized that the weakness of the Iowa Formula is in the difficulty of evaluation of e . He further attempted to verify the Formula by resolving the height of fill, H , in terms of thickness of metal, t , and diameter of pipe, D , using both the Iowa Formula and his own experience-proven empirical equation. Then he compared the results of the two equations. Shafer's empirical equation (15, p. 355) follows:

$$\Delta y = k_A \frac{H^{m'} D^{n'}}{t^{s'}} \quad \text{Eq. 4}$$

where Δy = vertical change in diameter

k_A = a constant

H = height of fill

D = diameter of pipe

t = thickness of metal

m' , n' , s' = exponents.

His table of comparisons shows that some values for height of fill, H , agree, but that many other values are in wild disagreement. He concluded that the Iowa Formula does not give proper value to certain component factors as warranted by experience.

Kelley (7, p. 364) arrived independently at a similar conclusion. He solved for height of fill, H , as a function of diameter of pipe, D , using the Iowa Formula. All other quantities in his equation were held constant, a typical value being used in each case. The result showed that H decreased as diameter, D , increased up to a specific diameter; but that above this specific diameter, H increased again. See Figure 45. Quoting Kelley (7, p. 365), "Such results are unreasonable. . ." and further, "It may be that the writer's (Kelley's) assumption of constant settlement ratio or a constant value of the modulus of passive resistance, or both, may be incorrect." The author is responsible for underlining modulus of passive resistance.

In the same year, 1948, Spangler (17, p. 249) reported the results of an attempt to apply his Iowa Formula to five sets of data from flexible pipe culvert tests in North Carolina. The objective was to compare the measured deflection against the deflection as calculated by the Iowa Formula. In each case e was calculated from the definition:

$$e = \frac{2h}{\Delta x}$$

In these tests a log of both h and Δx had been kept. The results satisfactorily confirmed the Iowa Formula, but the general observations regarding e as listed on page 13 were only partially confirmed. For example e was found to be independent of height of fill in some tests but not in all. Also there was some indication that e varied as the stiffness of the pipe wall, EI .

Since 1948 very little more has been done towards the evaluation of e ; consequently culvert designers have been slow to accept the Iowa Formula. In 1955 Spangler and Donovan (20) reported the investigation of a device designed to evaluate e directly for any given soil. The device was simply a circular rubber membrane in the side of a bin filled with soil of specified soil characteristics. Air pressure forced the rubber membrane against the soil. Deflection of the center of the member, as measured by a dial gage was considered to be $\frac{\Delta x}{2}$, and air pressure on the membrane (corrected for pressure of membrane inflation) was considered to be h .

The modulus of passive resistance was then calculated by means of the definition, $e = \frac{2h}{\Delta x}$. The experimental work was carried out as part of a master's thesis by James C. Donovan (6).

There appear to be some weaknesses and limitations in Donovan's method. In the first place the nearly spherical shape of the inflated membrane is nothing like the cylindrical surface of an actual pipe installation. Secondly, the average air pressure is not quite the same as the maximum horizontal pressure on the side of a flexible pipe. It is a logical assumption that a correction factor would be needed to convert Donovan's e to the e required in the Iowa Formula. Such a correction factor does not appear easy to determine. Thirdly, Donovan's investigation does not allow for an increase in vertical load on the soil as the membrane deflects. Actually if the loading of a culvert were simulated, the pressure on the membrane should increase at some rate which is a function of increasing vertical soil pressure. Consequently it is impossible by means of this method to determine whether height of fill has any effect on e . Finally, it is impossible by means of this method to measure the effect of stiffness of the pipe wall, EI . In order to investigate the characteristics of the modulus of passive resistance it is evident that a more powerful means is required than Donovan's device provides.

A paper was presented at the A.S.T.M. meetings in June

1957 by Russell E. Barnard (2) in an attempt to circumvent entirely the need for evaluating e . His proposed solution of the flexible pipe problem is highly theoretical and requires so many assumptions as to raise some question as to the results. It is as yet unproven.

C. Object and Scope of Investigation

The Iowa Formula appears to be acceptable as a rational approach to the flexible pipe culvert design problem where no rational approach has heretofore been available. This justifies a careful investigation of e because the rational approach, when properly substantiated by tests, leads to generally better design than does an equivalent empirical approach. In the first place the rational approach brings about a keener appreciation of the factors affecting performance. This in turn encourages greater care in design. In the second place, bounds and limitations of performance are more generally recognized. Extraordinary circumstances can be recognized and deliberated with confidence. Finally, the designer, being human, feels security in the use of a method if the principles are fully understood, for understanding reduces the problem to terms of his own experience. Such a method is more acceptable than the empirical approach which is based solely on the experience of someone else.

Quoting Shafer (15, p. 357):

Those engaged in the manufacture and distribution of flexible drainage structures see a definite need for a rational method of design. Not because design based on experience, such as the empirical equation, is wrong, but because it is possible that the full economy of flexible construction is not utilized in all cases. Furthermore, engineers prefer a rational approach to any problem, even though they use empirical methods in the solution of many problems. The rational approach also leads to a clearer understanding of the basic principles involved.

Besides being rational, the Iowa Formula includes pertinent factors which present-day design methods do not include. With an understanding of these factors, greater accuracy seems possible.

To the author it appears that the most satisfactory solution of the flexible pipe culvert design problem is by the use of the Iowa Formula for deflection, provided sufficient information can be developed regarding the modulus of passive resistance. The object of the project reported herein was to investigate the characteristics of the troublesome modulus of passive resistance, e , and to establish practical methods for evaluating it through a knowledge of its characteristics. Model study was the means adopted for the investigation.

The object as stated above is very general. It was necessary to delimit the scope within which the investigation proceeded. It is hoped, of course, that subsequent investigations will extend beyond the scope outlined here. There are so many factors which influence the modulus of passive

resistance, that it was impossible to completely describe the effects of all. This was particularly true with regard to the soil characteristics for soil is an infinitely variable material. The scope of the project excluded all but a few basic soil characteristics. Of the soil characteristics included, the difference between the effects of cohesionless soil (clean sand) and cohesive soil (clay) was considered to be of primary importance. Most soils are a combination of particle sizes varying between the limits of sand and clay. Gravel and boulders are usually encountered as bodies suspended in a matrix of finer particles and for most analyses may be sieved out and rejected. The effects of gradation of particle size were outside the scope of this project, but the effects of sand and clay certainly establish procedure and probably establish limits within which the effects of graded soils fall. A second basic soil characteristic which was investigated is compaction. Since compaction is not independent of soil density the two were investigated jointly. Other soil characteristics were generally considered to be outside the scope of the project except as they influence the basic soil characteristics indicated above. One example of such a soil characteristic is the moisture content which affects vertical soil pressure. Of course moisture content affects more than just soil pressure, but in any other respect it was outside the scope of this project. Such a limited

consideration of moisture content is not difficult to accept in the case of sand, for the behavior of wet and dry sand is the same (except for soil pressure effects) for most design purposes.

Clay poses a different problem. The behavior of wet and dry clays varies greatly. Nevertheless it seemed justifiable in this project to limit the investigation to the case of dry clay or nearly dry clay. If the moisture content exceeds the liquid limit the clay has negligible cohesion and the friction angle is very small. Such a case approaches hydrostatic pressure in culvert design, so the stresses on the pipe are all radial, and the deflection theory of failure does not apply. The modulus of passive resistance ceases to be a factor in such a case.

If the clay is in a plastic state, that is, if the moisture content exceeds the plastic limit, there is slight cohesion at the time of placement of the fill, and the internal friction angle is small. Under these conditions the pressure state still does not differ greatly from hydrostatic. Of course, in time cohesion will tend to develop, but critical load conditions occur at the time of construction before much cohesion has developed. Future investigation might well include the effect of moisture content on the deflection theory, but such an investigation was not included in the scope of this project.

In addition to the soil characteristics discussed above, the effects of the following factors were included in the scope of investigation: mean radius of the pipe, r ; stiffness of the pipe wall, EI ; and height of fill, H .

III. COMPARISON OF BASIC APPROACHES TO THE INVESTIGATION OF THE MODULUS OF PASSIVE RESISTANCE

All possible approaches to the investigation of e might be categorized in three areas: theoretical, full scale statistico-empirical, and model study. Actually model study is a combination of theoretical and statistico-empirical methods, but it is listed separately in order to distinguish model studies from full scale studies. Naturally most approaches will extend into more than one of the three areas just as model study extends into all three areas, but basically the categorization serves for comparison. The object of this chapter is to evaluate the three basic approaches and to show why model study is selected as the most promising approach to the investigation of e .

A. Theoretical Approach

A completely theoretical approach is virtually impossible at present. The modulus of passive resistance is highly dependent on soil characteristics, and too little is known about the physical chemistry of soils and surface phenomena to predict the modulus of passive resistance. Even if the chemistry of a soil were completely known, e could

not be predicted because it is also a function of other quantities such as radius of the pipe, boundary conditions including pipe distortion and foundation settlement, and degree and method of compaction if these are not accounted for as soil characteristics. If the soil were elastic, the influence of some of these factors might be theoretically computed, but in the case of large soil deformations as occur around a flexible pipe, the assumption of elasticity is questionable. For example, Williams (28) has demonstrated Marston's theory (8) that above a culvert in a homogeneous fill there may exist a horizontal plane (called the plane of equal settlement) above which the culvert deflection has no effect on relative soil displacement. Such a plane of equal settlement could not be arrived at by principles of elasticity if the soil were assumed to be a continuous elastic material. Rather with relatively large soil deformation, shear planes and shear regions form which break the continuity of the material.

Terzaghi has established a semi-theoretical approach for evaluating a quantity which he calls the coefficient of subgrade reaction (22, p. 297). He uses the notation, k , and defines it in exactly the same way as Spangler defines his modulus of passive resistance, e . The only apparent differences between k and e are the shape of the bearing surface and the way in which vertical soil pressure is applied. e is

defined for pipe surfaces, and k is defined for surfaces which are initially plane rectangles such as beams and slabs or sheet piling. In the case of beams and slabs (footings) which bear on a horizontal soil plane, Terzaghi uses the notation k_s and defines it as

$$k_s = \frac{q}{\Delta y}$$

where q is the increase in vertical load per unit area on the slab and Δy is the vertical displacement of the beam or slab due to load, q .

In the case of horizontally loaded piles, Terzaghi uses the notation, k_h , and refers to it as the coefficient of horizontal subgrade reaction.

$$k_h = \frac{h}{\Delta x}$$

where h is the increase in horizontal soil pressure against the piling and Δx is the horizontal displacement due to h .

At this point in his theory, Terzaghi makes three assumptions, the first two of which are deduced from empirical observations.

(A) For cohesionless material, such as clean sand, k_h increases with depth according to the relationship

$$k_h = m_h H$$

where m_h is a constant of proportionality and H is the depth below the surface.

(B) For stiff clay, k_h is independent of the depth

below the surface.

(C) k_h is independent of horizontal pressure, h .

If these assumptions are accepted, Terzaghi's theory for k_h in stiff clay is precisely the same as Spangler's theory for e in any non-saturated soil. See page 13. Terzaghi's assumption that k_h is independent of depth below the surface is equivalent to Spangler's suggestion that e is independent of height of fill. Terzaghi's assumption that k_h is independent of h is equivalent to Spangler's use of e in the derivation of the Iowa Formula wherein it is assumed that e is independent of Δx and h (3, p. 28). One serious difference shows up between Terzaghi's Assumption A regarding sand, i.e., k_h is a function of height of fill, $k_h = m_h H$, and Spangler's observation that e is independent of height of fill in all soils. This difference must not be overlooked in the case of sand; but in order to complete this theoretical discussion of e according to Terzaghi's theory, his assumptions are here rewritten as they would apply to the flexible culvert situation.

(A) For cohesionless material, such as clean sand,

$$e = mH \qquad \text{Eq. 5}$$

where H = height of fill,

m = constant of proportionality.

(B) For stiff clay e is independent of the height of fill.

(C) e is independent of horizontal pressure, h , in all soils.

The agreement between Spangler's theory and Terzaghi's theory as applied to culverts in clay suggests that for stiff clay the above listed assumptions regarding the use of e are acceptable. Accordingly, all that is needed to evaluate e for any stiff clay is a single test by some kind of a device which will exert a lateral pressure against a typical soil sample (at proper moisture content, proper compaction, etc.) and at the same time measure the horizontal displacement. Of course the pressure must be exerted by a cylindrical plate shaped like the side of a pipe; and the pressure, h , and the deflection, Δx , must be measured at the horizontal diameter of the plate. Ideally this cylindrical plate should deflect from a circular arc to some unknown arc just as the pipe section would, but since the deviation is of secondary importance for small deflections a device as shown in Figure 3 might achieve sufficiently accurate results. See Appendix B. The device shown is hereinafter referred to as the Modpares Device (modulus of passive resistance device). Basically the device applies lateral pressure, h , to the soil by means of a membrane which is inflated by air pressure. The air pressure, corrected by some tare amount required to deflect the membrane alone, becomes h . The displacement $\frac{\Delta x}{2}$ may be measured by a dial gage as shown. The required angle of

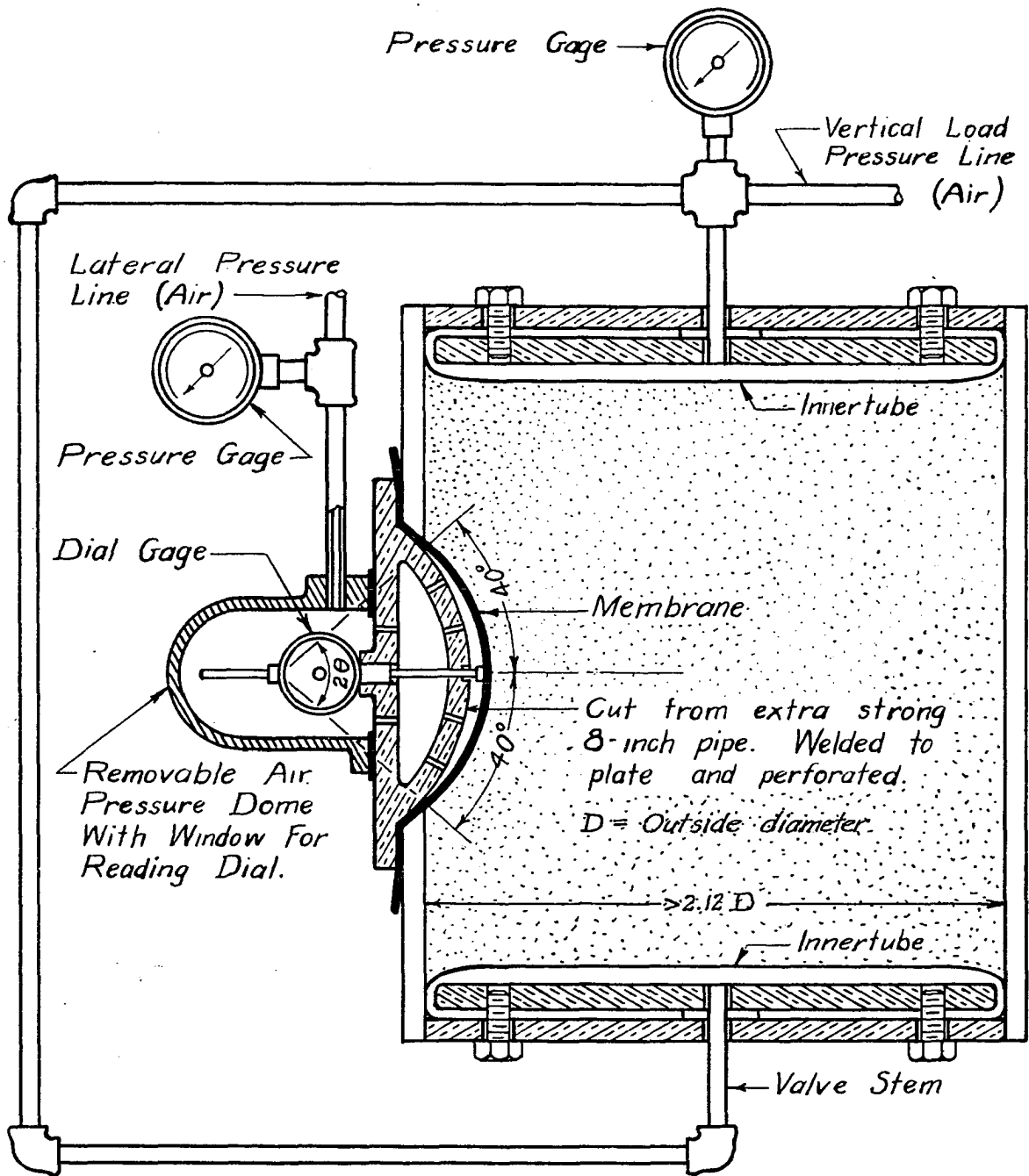


Figure 3. Diagrammatic sketch of proposed Modpares device to be used in determination of the modulus of passive resistance of a soil sample

contact, 2θ , and the shape of the inflated membrane as a bearing surface are discussed in Appendices A and B. The width of the bin, B , is indicated as being greater than $2.12D$. This value is based on Terzaghi's assertion (22, p. 316), that the distance to the effective boundary of soil displacement is three times the width of the bearing surface, B . The width of the bearing surface in this case is equal to or less than $2\left(\frac{D}{2}\right) \sin 45^\circ$ as demonstrated in Appendix A; so

$$B \leq 0.707D.$$

This effective boundary of soil displacement is based on the theory of elasticity as applied by Terzaghi (23, p. 424). If this Modpares device were developed, the results would be subject to all of the above mentioned theoretical assumptions.

As shown in Figure 3 the Modpares device is equipped with a means for applying vertical load to the soil specimen. When used with stiff clay, the clay must be compacted and consolidated to the same degree called for in the field, but as described in the above paragraph, e is assumed to be independent of height of fill (or vertical pressure) for stiff clay; so vertical pressure should have no effect on the test after consolidation is completed.

According to Terzaghi's Assumption A, sand presents a different problem, since e is directly proportional to the height of fill. In order to use the Modpares device for

sand, it would be necessary to run two or more tests. In each test a different vertical load would be applied to the soil. A graph of h versus $\frac{\Delta x}{2}$ could be plotted for each test. The slope of the best fit straight line would become the modulus of passive resistance, e , for each; then assuming a linear variation of e as a function of height of fill the value of m could be solved in the equation, $e = mH$. Thus e may be determined for sand as well as stiff clay by the Modpares device. After this device was designed a similar device was proposed by Dr. J. M. Hvorslev¹ in a recent as-yet-unpublished discussion by Brown of a paper by Spangler and Donovan (20), enclosed in a letter from Turnbull² to Burggraf.

Terzaghi attempts to circumvent the need for a device to measure e by providing tables which give typical values of e for various types of soil, at various degrees of compaction, and for a 1 ft. square bearing plate. Of course his tables

¹Consultant, Soils Division, U. S. Army Engineer Waterways Experiment Station, C. E. Vicksburg, Miss.

²Turnbull, W. J., Engineer, Chief, Soils Division, U. S. Army Engineer Waterways Experiment Station, C. E. Vicksburg, Miss., in a letter to Fred Burggraf, Director, Highway Research Board, as noted in a copy received by M. G. Spangler, with enclosure by Donald N. Brown, Engineer, Soils Division, U. S. Army Engineer Waterways Experiment Station, C. E., Vicksburg, Miss. "Comments on 'application of the modulus of passive resistance of soil in the design of flexible pipe culverts' by M. G. Spangler and James C. Donovan." Private communication, 25 July 1957.

are designed especially for horizontally loaded piles, not culverts.

The question arises as to the degree of error incurred by the basic assumptions in the above semi-theoretical discussion. Regarding Assumption A that $e = mH$ for sand, Figure 4a shows a probable plot of h versus H for average, clean sand according to Terzaghi (22, p. 324). The horizontal displacement $\frac{\Delta x}{2}$ is constant for all heights of fill, H . Since

$$e = \frac{2d(h)}{d(\Delta x)}, \text{ then } d(h) = H m \frac{d(\Delta x)}{2}.$$

Integrating for a given height of fill,

$$h = m H \frac{\Delta x}{2}.$$

It is assumed that $\frac{\Delta x}{2} = 0$ when $h = 0$. Since $\frac{\Delta x}{2}$ is constant for all heights of fill as specified for construction of the figure, the plot of h versus H according to Assumption A is a straight line as shown dotted. The discrepancy between the probable and the theoretical plots is significant.

Regarding Assumption B that e is independent of height of fill for stiff clay; if $\frac{\Delta x}{2}$ is constant, the h versus H plot should be a straight vertical line as shown dotted in Figure 4b. The same figure shows the qualitative discrepancy between the theoretical and a probable plot as proposed by Terzaghi (22, p. 324).

Regarding Assumption C that e is independent of horizontal pressure, h ; there is considerable discrepancy between

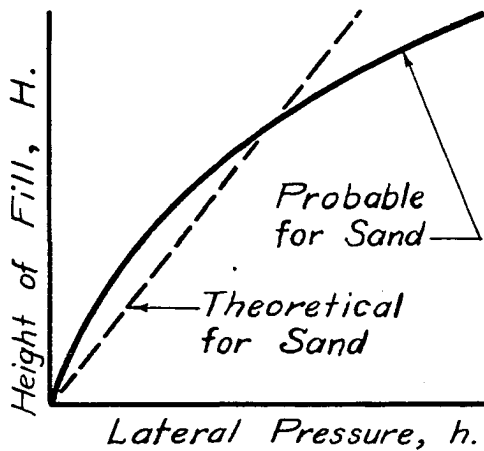


Figure 4a. Probable and theoretical values for lateral pressure as a function of height of fill for sand; deflection, Δx , is constant

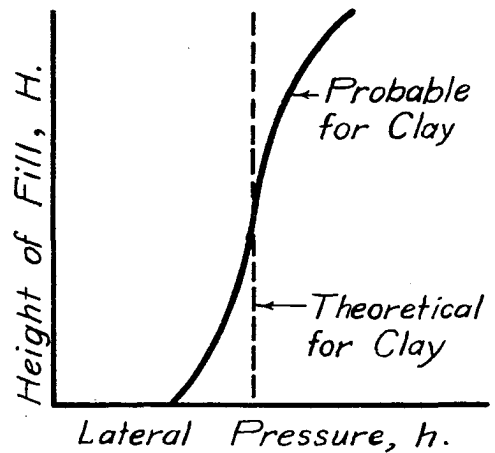


Figure 4b. Probable and theoretical values for lateral pressure as a function of height of fill for stiff clay

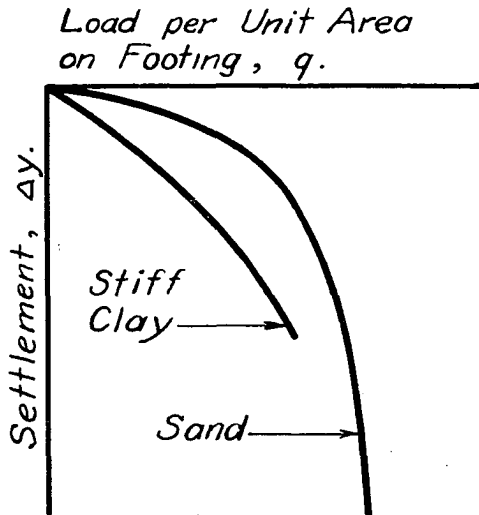


Figure 4c. Load-settlement diagrams for footings on typical sand and stiff clay, after Taylor (21, p. 576)

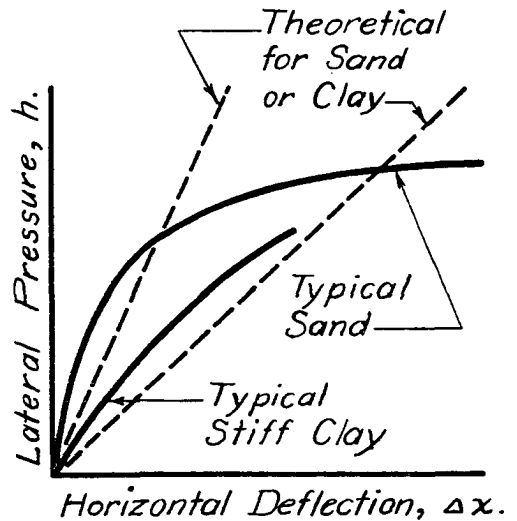


Figure 4d. Lateral pressure displacement diagrams for vertical bearing surfaces in sand and clay

typical measured values according to Taylor (21, p. 576) and theoretical values if the closely related case of penetration of a bearing plate is considered. Figure 4c shows an average plot of settlement versus load stress, q , for any soil.

Transposed into the horizontal equivalent the typical measured plots of h versus Δx appear as shown in Figure 4d for sand and stiff clay where H is equivalent to q and Δx replaces Δy . At the same time the theoretical plot is computed from Assumption C:

$$e = \frac{2d(h)}{d(\Delta x)} = \text{Constant. Integrating,}$$

$$h = m' \Delta x \quad \text{Eq. 6}$$

where m' is a constant and where $\Delta x = 0$ when $h = 0$. A plot of this curve is shown dotted in Figure 4d. Note that the discrepancy with typical measured values is sizeable.

This consideration of error is only qualitative, but it does point out some of the weaknesses in the adaptation of Terzaghi's semi-theoretical approach to the investigation of e .

B. Full Scale Statistico-Empirical Approach

The full-scale statistico-empirical approach would seem to be the most logical approach if judged by present methods of design of flexible pipe culverts. Nearly all design at the present time is based on average performance of

installations in the field (1). The adjective, "full-scale" is used to distinguish the study of actual pipe installations from model study as described in the next section. "Full-scale" includes not only test installations but service installations as well. The coined adjective "statistico-empirical" refers to the empirical development of statistical data from numerous installations. For example logs of deflections and soil pressures on many installations might be assembled. The influence of various factors such as type of soil and degree of compaction could then be arrived at by statistical methods. Very likely a digital computer would be needed to solve the simultaneous equations involved for the number of unknowns would be equal to the total number of independent soil characteristics plus the total number of independent pipe characteristics plus the total number of boundary conditions. Because of so many unknowns, a tremendously large number of installations would have to be tested at a very high cost. As a matter of fact it is doubtful that enough pipe culverts are installed in a year in this country to adequately analyze all of the variables that influence e.

In addition, a very complicated arrangement of pressure cells, settlement plates and deflection gages would have to be designed, installed and maintained. Also a very elaborate soil testing and inspecting program would be imperative.

Time as well as money would be required. In addition to the time required for installation of measuring equipment, many months of time might be required to investigate the time lag factor after each increment of fill in certain soils. Most contractors would not stand for such a delay during construction of the fill. Moreover, of the culverts that are installed, many are oversized, so the performance up to failure conditions cannot be observed.

A more productive approach would be to set up an elaborate series of full scale test installations in which certain factors could be controlled and in which tests could be conducted on up to the point of failure. But even under these circumstances serious problems would arise. First the project would be very expensive. Not only would hundreds of installations be required, but each installation would probably have to be housed to preserve a controlled moisture content. Expensive sieved and graded fill would be required, and to include all present day fills a height of approximately 200 feet would be necessary.

Another serious problem concerns the accuracy of soil pressure cells. No pressure cells in use at present are entirely satisfactory. In order to accurately measure soil stresses the pressure gage would have to distort on its surface exactly the same as the displacement pattern which the soil would assume if the gage were replaced by the original soil. Moreover, the friction angle between pressure cell and

soil would have to be the same as the internal friction angle of the soil; and the cohesion between the pressure cell and the soil would have to be the same as cohesion within the soil. Such a gage is virtually impossible. Based on years of experience in measuring stresses on culverts, Spangler still places more faith in the simple friction ribbon than in soil pressure cells. The friction ribbon is a stainless steel ribbon (18, p. 33) covered with a sheath and embedded in soil. The amount of force required to just start pulling the ribbon through the sheath is recorded. By proper pre-calibration of the coefficient of friction of the sheath against the ribbon, the normal soil pressures against the ribbon can be evaluated. Time and moisture content affect the accuracy of the ribbon just as they affect the accuracy of most soil pressure cells (18, p. 51).

The above discussion presupposes that e is computed from its definition, $e = \frac{2d(h)}{d(\Delta x)}$, where the lateral soil pressure, h , and deflection of the pipe, Δx , must both be measured. A more indirect method might be employed by which the hard-to-measure h would be eliminated. Spangler's Iowa Formula could be rewritten solving for e . The result follows:

$$e = 16.4 K \frac{W_c}{r \Delta x} = 16.4 \frac{EI}{r^4} . \quad \text{Eq. 7}$$

Obviously values must be assumed or evaluated for K and W_c which are so questionable as to make the entire approach undesirable.

In the final analysis the full scale statistico-empirical approach becomes so clumsy as to make it absurd. The sole objective is to evaluate e so that the deflection of the pipe, Δx , might be predicted. Yet it is necessary to measure Δx in order to evaluate e . If enough measurements of Δx could be made in full scale installations to accurately determine e for all cases, there would be no further need for e because the deflections would be known.

C. Model Study Approach

The most promising approach to the evaluation of modulus of passive resistance, e , is model study. As shown in Section IV all of the design conditions for a true model can be met reasonably well so that questionable assumptions need not be imposed upon the investigation. See the assumptions listed on page 25. This makes it possible by model analysis to study the effects on e of such quantities as radius of pipe, r , height of fill, H , stiffness of pipe wall, EI , etc. Despite the greater power of the model study, no more time and money is required to run a series of tests than would be required to run an equivalent series of tests on the Donovan Device or Modpares Device. Certainly much less time and money would be required than for full scale tests. Finally by model analysis the control is improved to the point where almost any quantity which affects e may be isolated and studied

separately. Thus the work can be carried along on an economical basis at the convenience of the researcher. The next chapter shows a development of principles of model study as used in this investigation.

The basic approach in the next section follows very closely the development of general principles of similitude according to Murphy (10). Rocha (13) has proposed some basic principles of similitude specifically for soil with some of the same results observed in this project, but Murphy's development is more general and makes possible the investigation of many factors which Rocha has neglected.

IV. THEORY OF MODELS AS APPLIED TO FLEXIBLE PIPE CULVERTS
UNDER EARTH FILL

A. Basic Principles

The theory of true models is conveniently arrived at through consideration of a generalized Π -term relationship which describes the performance of a system called the prototype. Such a relationship follows:

$$\Pi_1 = F(\Pi_2, \Pi_3, \Pi_4, \dots, \Pi_s) \quad \text{Eq. 8}$$

where Π_1 is a function of Π_2, Π_3, Π_4 , etc. In this generalized form each Π -term is dimensionless, is independent of all other Π -terms, and represents one or more of the primary quantities which affect the system. Now since Equation 8 is dimensionless it is perfectly general and applies equally well to any other system which is a function of the same variables regardless of the units of measurement and regardless of the magnitudes of the measured quantities. Specifically it applies to a model system for which variables are the same (10, p. 58). Using the subscript, m , for the model, the Π -term relationship becomes:

$$\Pi_{1m} = F(\Pi_{2m}, \Pi_{3m}, \Pi_{4m}, \dots, \Pi_{sm}) \quad \text{Eq. 9}$$

Since Equations 8 and 9 describe the same phenomenon, since the forms of both equations are the same, and since all

Π -terms are dimensionless, corresponding Π -terms may be equated. The equations of the corresponding Π -terms provide a set of design conditions for creating a model. These design conditions are:

$$\Pi_{2_m} = \Pi_2$$

$$\Pi_{3_m} = \Pi_3$$

$$\Pi_{4_m} = \Pi_4$$

$$\vdots$$

$$\Pi_{s_m} = \Pi_s$$

The equation of the secondary Π -terms is a prediction equation, $\Pi_1 = \Pi_{1_m}$.

Of course, any other Π -term of the general Π -term relationship might be used instead of Π_1 as the basis for a prediction equation since all Π -terms are independent. In such a case the equation, $\Pi_1 = \Pi_{1_m}$, would become a design condition.

B. Development of Design Conditions and Prediction Equations

It was mentioned above that each Π -term is a dimensionless quantity which includes one or more of the primary quantities that affect the system. This provides a starting point from which a general Π -term relationship may be written

for the system. In this investigation it is proposed that the modulus of passive resistance, e , be studied. The system consists of a flexible pipe culvert (without internal pressure) under an earth fill. As defined before, $e = 2 \frac{d(h)}{d(\Delta x)}$ where $h =$ horizontal soil pressure against the side of the pipe on the horizontal diameter, and $\Delta x =$ the increase in horizontal diameter of the pipe. As explained in the preceding section h cannot be easily evaluated so it may be replaced by other primary quantities on which it depends. All independent primary quantities which appear to influence e must be listed. A reasonable set of such primary quantities follows:

1. $e =$ modulus of passive resistance FL⁻³
2. $r =$ mean radius of the culvert L
3. $\lambda =$ any other pertinent dimension in the soil L
4. $EI =$ stiffness factor for the wall of the pipe FL²
5. $p_v =$ vertical soil pressure at any depth, z ,
in soil FL⁻²
6. $e_v =$ void ratio of the soil (function of
density)
7. $w =$ water content of the soil (per cent)
8. $\phi =$ internal friction angle of the soil
9. $c =$ cohesion of the soil FL⁻²
10. $\Omega =$ compactive effort (work per unit volume) FL⁻²
11. $l =$ length of the pipe (may be included in λ) L

Quantities 5 to 10 are soil characteristics. The bedding angle, α , is not listed in the above set because it can be written in terms of λ . See Figure 2. λ also covers such primary quantities as configurations of boundaries, displacements of the boundaries, and displacement (or strain) at any point in the soil. λ is considered separately rather than in connection with λ in order that two dimensional stress condition might be justified later.

Now the above set of primary quantities may be arranged into a general Π -term relationship. This can be done arbitrarily just so the Π -terms are all dimensionless and independent of each other and just so all of the primary quantities are included. The minimum number of Π -terms required is determined by the Buckingham Pi-theorem (10, p. 36) to be nine. The Buckingham Pi-theorem states in effect that the required number of Π -terms is equal to the number of primary quantities (11 in this case) minus the number of dimensions in which these quantities are measured (2 in this case--F and L). One possible Π -term relationship may be written as follows:

$$\frac{er}{\Omega} = f \left(\frac{\lambda}{r}, \frac{EI}{\Omega r^4}, \frac{P_V}{\Omega}, e_V, w, \phi, \frac{c}{\Omega}, \frac{l}{r} \right) .$$

The same relationship would obtain if the primary quantities were written in terms of a unit of length instead of total length of the pipe except that the $\frac{l}{r}$ term would

disappear. For instance in terms of unit length of the pipe the dimensions of e , EI , p_v , c , and Ω are dimensionally altered as follows:

<u>Primary quantity</u>	<u>Dimensions (per unit length of pipe)</u>
e	FL^{-4}
EI	FL
p_v	FL^{-3}
c	FL^{-3}
Ω	FL^{-3}

That these dimensions do not affect the dimensionless quality of the π -terms is easily checked. The π_1 -term is

$$\pi_1 = \frac{er}{\Omega} = \frac{FL^{-4}L}{FL^{-3}} = 1 \quad \text{etc.}$$

Since l is now a fixed unit length, it can no longer be included as one of the 11 primary quantities, and only 10 primary quantities are available. The Buckingham Pi-theorem sets the number of π -terms at 8 instead of the 9 shown in Equation 10. Inspection of the last π -term, $\frac{l}{r}$, reveals that it is no longer an independent π -term since l is now fixed and r occurs in other π -terms. The $\frac{l}{r}$ term is thus eliminated.

The above rational attempt to delete the $\frac{l}{r}$ term accomplishes nothing more than imposition of the assumption that a two dimensional stress (or strain) state exists. This assumption would require that there be no relative displacement of any point of the system in the direction of the pipe axis. Of

course relative displacement of the soil does occur in the direction of the pipe axis. Indeed, many soil fills expand visibly with respect to the pipe in the direction of the pipe axis as the height of fill is raised or as a surcharge load is applied; but experience generally indicates that within the accuracy of present-day design methods, the assumption of two-dimensional stress (strain) is reasonable.

Nevertheless, it must be pointed out here that the general Π -term relationship is not limited to a two-dimensional stress (strain) state. All that is necessary to achieve three dimensional stress is to include the ninth Π -term, $\frac{l}{r}$. Very likely the time will come when sufficient accuracy can be developed in predicting the performance of a pipe-fill system to justify inclusion of three dimensional stress. Within the scope of this study, however, only two dimensional stress (strain) will be considered.

C. Basic Design of Model

With the Π -term relationship established the model can be designed by equating corresponding Π -terms for the model and the prototype. These particular design conditions are here referred to as Design Conditions I.

Design Conditions I Let the scale factor be $n = \frac{r}{r_m}$.

1. $\lambda_m = \frac{\lambda}{n}$ Geometrical similarity must exist

throughout the soil of both systems. This condition establishes all boundaries in the soil of the model system such as the pipe boundary, rock ledges, wing walls, bed rock, etc. Actually the soil surface should be included as a boundary, but an equivalent stress boundary may be substituted as described later. This condition also establishes the dimensions and positions of zones of differing soil properties.

$$2. (EI)_m = \frac{(EI)}{n^4}$$

This gives the required stiffness factor for the wall of the model pipe.

$$3. (p_v)_m = p_v$$

$$\Omega_m = \Omega$$

$$(e_v)_m = e_v$$

$$w_m = w$$

$$\phi_m = \phi$$

$$c_m = c$$

These conditions indicate that all soil characteristics in the model must be the same as the corresponding soil characteristics in the prototype.

Since the purpose of this model study is to determine the modulus of passive resistance, e , the equating of the first π -term for model to the first for prototype provides a prediction equation. It is here referred to as Prediction

Equation I.

Prediction Equation I

$$\frac{er}{\Omega} = \frac{e_m r_m}{\Omega_m}$$

or since $\Omega = \Omega_m$ and $n = \frac{r}{r_m}$ from the design conditions, the prediction equation becomes:

$$e = \frac{e_m}{n}$$

Apparently e is not a property of the soil characteristics alone, for it is not constant in model and prototype as soil characteristics were assumed to be. But rather e is a function of r ; that is, $er = e_m r_m$; or er remains constant for any given set of soil characteristics. This result is very important.

Those who have worked with models might be skeptical of the third design condition regarding constant soil characteristics, because so often the model must be constructed of a different material than the prototype. This is particularly true of the unit weight of the material which must often be n times as heavy as the prototype. In Design Conditions I the unit weight of the soil, γ , did not appear. Rather a soil pressure, p_v , which is a function of the unit weight was used. That eliminated the use of unit weight since all primary quantities must be independent. In order to investigate the design conditions including unit weight of the soil, it

is a simple matter to replace the soil pressure, p_v , by the primary quantities which determine p_v ; namely, height of fill, H , and unit weight, γ , of the soil. In this analysis any superimposed loads other than soil loads are neglected. One possible Π -term relationship follows:

$$\frac{er}{\Omega} = f' \left(\frac{\lambda}{r}, \frac{H}{r}, \frac{EI}{\Omega r^4}, e_v, w, \frac{c}{\Omega}, \phi, \frac{\gamma r}{\Omega} \right)$$

Based on this modified Π -term relationship, a new set of design conditions are listed below. Again the subscript m refers to the model.

Design Conditions II

Let n be the length scale factor, i.e.,

$$n = \frac{r}{r_m} .$$

$$1. \quad \lambda_m = \frac{\lambda}{n}$$

$$H_m = \frac{H}{n}$$

Geometrical similarity must exist throughout the soil model.

$$2. \quad (EI)_m = \frac{EI}{n^4}$$

This gives the stiffness factor required for the wall of the model pipe.

$$3. \quad (e_v)_m = e_v$$

$$w_m = w$$

$$c_m = c$$

$$\Omega_m = \Omega$$

$$\phi_m = \phi$$

$$\gamma_m = n\gamma$$

These conditions indicate that all soil characteristics in the model must be the same as the corresponding soil characteristics in the prototype except the unit weight.

Prediction Equation II

$$\frac{e r}{\Omega} = \frac{e_m r_m}{\Omega_m} \quad \text{or} \quad e = \frac{e_m}{n}. \quad \text{This is the same as}$$

Prediction Equation I.

From the first design condition above, geometrical similarity is confirmed. From the second design condition, $(EI)_m = \frac{EI}{n^4}$ is confirmed. But contrary to the findings of Design Conditions I the soil characteristics cannot be held constant because Design Condition 3 requires that the unit weight of the soil in the model be n times the unit weight of the soil in the prototype. This is impractical, but even if it were accomplished it is doubtful that all other soil characteristics could be held constant as required in both model and prototype. Herein lies the major limitation to the use of model study of a pipe-fill system. For further analysis in this paper, Design Conditions I will be used, but it should be remembered that the unit weight of the soil must be considered in order to establish values for p_v . In order to accurately develop p_{v_m} (or γ_m) in a model in which all soil characteristics are the same as in the prototype, it would be necessary to superimpose additional gravity forces on the soil of the model. It might be possible to place the model in a centrifuge or hang weights on pins at different levels of the model as was done in a model analysis of a cantilever section of the Hoover Dam (26). Either of these methods shows considerable promise.

Or as an alternative method the additional gravity forces might be achieved for any given elevation in the soil by merely superimposing an appropriate surface load on the soil in the model such that p_{v_m} at the level of concern is the same as p_v at the corresponding level in the prototype. Unfortunately p_{v_m} at any other level in the model would not be the same as the corresponding p_v in the prototype. For this investigation, however, it was found sufficiently accurate to neglect the variation of p_{v_m} throughout the height of the model fill, and to superimpose a surface load which develops the proper p_{v_m} at the level of the top of the pipe. By way of limits it is reasonable to suspect that the variable p_{v_m} would cause greater inaccuracy if the pipe were very light weight and the fill very low. This follows from the fact that the per cent variation in p_{v_m} from the top to the bottom of the pipe is greater for a low fill. Such reasoning accounts for the fact that this project is limited to high fills. A definition of high fill is given in conclusions of this dissertation.

Verifications of these design conditions and prediction equations are given in Appendices C and D. Appendix C is a rational demonstration of the prediction equation by simple methods of strength of materials. Appendix D is a verification of the design conditions using principles of elasticity.

Two important conclusions can be drawn from the

foregoing discussion:

1. Model study appears to be a powerful method not only for investigating e but for investigating the deflection of a pipe directly should the economics of a particular installation justify a model study.

2. In two geometrically similar soil systems with the same soil characteristics and soil pressures, e_r is the same. That is

$$e_r = e_m r_m \cdot \quad \text{Eq. 11}$$

This statement does not preclude the possibility that e_r might vary as a function of other quantities. For example, tests showed that e_r varies as a function of compactive effort, Ω , and soil type.

Physical verification of Equation 11 above was easily accomplished and is reported in subsequent sections. Since e_r rather than e is constant for a given set of soil characteristics, there is good reason to proceed with e_r as a modulus rather than e . It is proposed for the purposes of this report that e_r be referred to as the modulus of soil reaction. The use of e_r does not affect the Iowa Formula in any way. As a matter of fact it has already been written in terms of e_r in this dissertation. See Formula 1.

V. EXPERIMENTAL PROCEDURE

A. Construction of the Model Cells

In the construction of the models, two-dimensional stress conditions were assumed. This assumption made it possible to basically enclose a short model section of flexible pipe lengthwise between two rigid, frictionless, plane boundaries. In order to make those boundaries as nearly rigid as possible heavy construction was employed. Except for this precaution the yield of the boundaries was ignored. Friction was not eliminated, but a method of compensating for friction loss was developed. Two boundary devices were constructed of different sizes. They are referred to as model cells. See Figures 11 and 13.

The first major problem of detail was the over-all dimensions. The smaller model cell was so planned that it would serve a number of purposes. One purpose was the investigation of boundaries of relative soil displacement perpendicular to the pipe axis by X-raying the soil as the load was increased. Ideally, a separate model with geometrically similar boundaries should be constructed for each proposed field installation, but with a very few exceptions culvert projects do not justify such an elaborate analysis.

Furthermore the resistance of the foundation material to pressures imparted by the fill is practically indeterminate. A more reasonable approach is to search for any practical boundaries of relative soil displacement due to the deflection of the pipe. The term relative here refers to the relative displacement of the soil in the region around the pipe with respect to the displacement of the soil in the same region if there were no pipe but only continuous soil under the same loading conditions. If boundaries of relative soil displacement could be located, a model cell might be constructed which would at least include these boundaries. To facilitate discussion the following nomenclature is used. The Z-boundaries are planes perpendicular to the z-axis or pipe axis. The X-boundaries and Y-boundaries are planes perpendicular to the x and y axes respectively where the x-axis is horizontal and the y-axis is vertical. See Figure 5a.

Z-boundaries

Since two dimensional stress conditions were assumed the Z-boundaries could theoretically be spaced arbitrarily. From a practical standpoint, the greater the spacing of the Z-boundaries, the less would be the influence of wall friction and the more accessible would be the cell. On the other hand, the capacity for making well defined X-rays

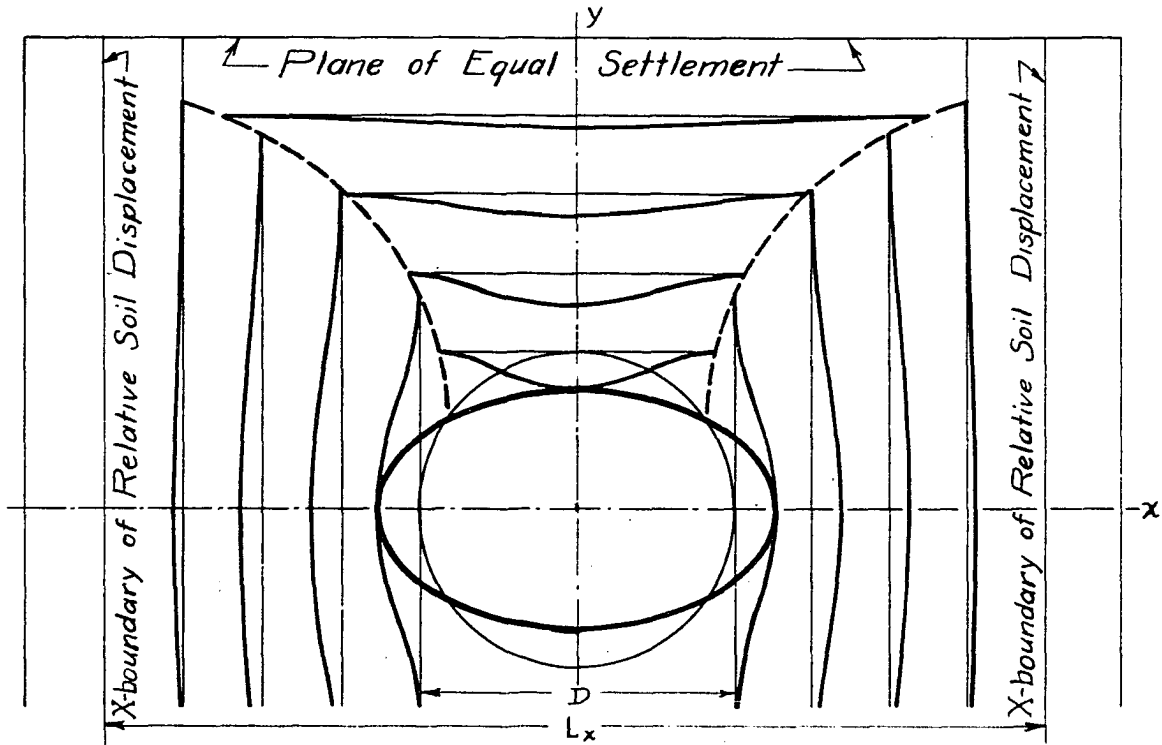


Figure 5a. Relative soil displacement-vertical components above the pipe and horizontal components at the sides

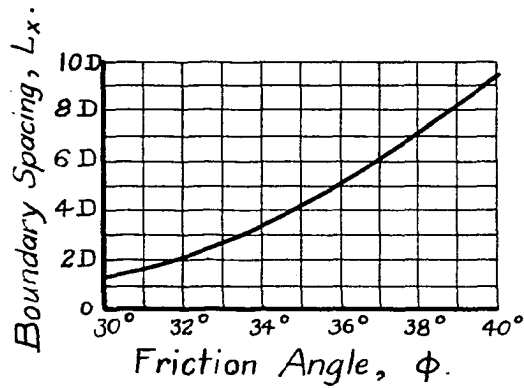


Figure 5b. Spacing of X-boundaries of relative soil displacement by Peck Theory

decreases with increased spacing of the Z-boundaries. A spacing of 2.5 inch was finally accepted since it is the minimum width within which a standard Proctor hammer can be operated.

X-boundaries

The X-boundaries posed a more difficult problem. Attempts to apply commonly accepted stress theories lead to variable results. For example, if the soil were assumed to be elastic, the X-boundaries would be at infinity. In such a case there would be no choice but to design the boundaries of the model geometrically similar to the prototype. According to a popular theory of bearing loads on soil as described by Peck (11), spacing of the X-boundaries is calculated to be at least $2.5D$ for saturated, consolidated clays and $2D$ to $9.5D$ for sand. D is the pipe diameter. See Appendix E for computations. The great variation in the X-boundaries for sand is a function of the internal friction angle. The adaptation of Peck's footing theory to culverts is not entirely unquestionable, but for lack of a more rational concept, the results of this theory for sand are shown in Figure 5b, where L_x = the distance between X-boundaries. Since the X-boundaries for sand vary so much, it was arbitrarily decided to design the smaller model such that the distance, L_x , was about $4D$. It might then be possible to

observe the effect if the internal friction angle, ϕ , increased so much that the boundaries of relative displacement in the soil exceeded L_x . At this point one weakness in the use of Peck's bearing method becomes apparent. The boundary of relative soil displacement increases as the friction angle increases. See Figure 5b. But the friction angle increases as the density of the sand increases. As the pipe expands laterally under an increasing height of fill, the soil adjacent to the pipe becomes denser, so the spacing of the X-boundaries should increase. Peck's theory assumes that the boundaries are fixed.

In the design of the model, an L_x of $16 \frac{3}{8}$ inches was finally decided upon since the maximum size of available X-ray film was 14 inches x 17 inches and the outside dimensions of the corresponding cassette were $14 \frac{7}{8}$ inches x $17 \frac{7}{8}$ inches. By using $\frac{3}{4}$ inch thick steel for the sides of the model, L_x was fixed at $16 \frac{3}{8}$ inches, i.e., $17 \frac{7}{8}$ inches less $1\frac{1}{2}$ inches for the steel sides. Culvert model sections were cut from tin cans with an average diameter of $3 \frac{7}{8}$ inches. The final relationship of L_x to D was then $L_x = 4.2D$.

Y-boundaries

The $14 \frac{7}{8}$ inches x $17 \frac{7}{8}$ inches X-ray cassette automatically fixed the spacing of the Y-boundaries of the model

at $14 \frac{7}{8}$ inches less the thickness of the steel sides or $13 \frac{5}{8}$ inches. For most of the model studies the pipe section was placed in the center of the model cell with fill soil of uniform characteristics completely surrounding it. The resulting relative deflection of the pipe with respect to adjacent soil was symmetrical about the horizontal diameter as an axis. This is not in complete agreement with common culvert design principles. Ordinarily a bedding angle, α , is specified as the bearing surface for the bottom of the pipe, while a uniform load the width of the pipe is assumed to act on top. According to the Iowa Formula, the coefficient, K, is dependent on the bedding angle. As the bedding angle varies from 0° to 90° , the coefficient K varies from 0.110 to 0.083 according to Spangler (18, p. 29). His table of values is given in Table 1.

Table 1. Bedding constants

Bedding angle, α , degrees	Bedding constant, K
0	0.110
15	0.108
30	0.102
45	0.096
60	0.090
90	0.083

The bedding problem is particularly important in connection with relatively stiff pipes. As flexible pipes deform the bedding angle quickly approaches 90° in installations which have been carefully backfilled. But even without care, if the bedding angle should fall short of 90° , it is evident from Table 1 that the corresponding values for K would not differ greatly. It appears that Kelley (7, p. 364) is justified in conservatively assuming a value of $K = 0.1$ for flexible pipe culverts in general. Now if the bedding angle is 90° the pipe is actually surrounded by fill soil in field installations, and a similar situation is justified in the model.

Further justification for placing the model pipe in the center of the cell follows from a description of recommended installation procedures.

1. The culvert site is cleared of trash and plant growth and excavated to a satisfactory sub base.
2. Selected fill material is placed in the site and compacted to a height at least equal to one pipe diameter above the top of the proposed pipe.
3. A ditch is excavated for placement of the pipe. A bedding is formed, the pipe is installed, and soil is carefully compacted around the pipe to the top of the pipe. Loose fill is placed in the trench to the level of the original compacted fill.

4. The fill, compacted or random, is then continued on up.

Since all unsatisfactory sub-base soil is removed and replaced by selected fill, the pipe is completely surrounded by fill soil of generally uniform characteristics. Under such conditions the relative displacement of the pipe with respect to surrounding soil is essentially symmetrical about the horizontal diameter as an axis. If the foundation soil is less compressive than the fill soil this assumption of vertical symmetry gives conservatively larger calculated deflections; and if the foundation soil is more compressive than the fill, the sub base soil would not be classified as satisfactory. Again it appears justifiable to place the model pipe in the center of the cell.

With a $3 \frac{7}{8}$ inch model pipe in the center of the cell, soil coverage in the y-direction was about $4 \frac{3}{4}$ inches from the top edge of the pipe to the Y-boundary of the cell. It became apparent during tests that this boundary was not outside of the zone of relative soil displacement throughout the entire range of pipe deflection. The upper limit of relative soil displacement is simply the definition of plane of equal settlement as used by the Marston theory (16, p. 417). Consequently the Y-boundary spacing should be such that the plane of equal settlement lies within the model cell. Spangler (16, p. 423) published an equation for calculating

the soil coverage, H_0 , from the top of the pipe to the plane of equal settlement. Solutions of the equation in terms of pipe diameter, D , may be picked from a graph by Spangler (16, p. 424) which is reproduced here as Figure 14c. H_0 is the value of H at the points of intersection of the incomplete ditch condition lines with the complete ditch condition line. According to the range of values plotted, H_0 may vary from 0 to about $8D$. High values are improbable in the case of pipe deflections of 5 per cent or less, but the large range of values demands that the effect of Y-boundaries be checked. This was accomplished by means of the large model on which the boundary spacing was greater.

The large model cell was so constructed that all linear dimensions were twice as large as the small cell. See Figure 13. The Z-boundaries were constructed of 2 inch x 6 inch tongue and groove fir with an 18 gage galvanized iron liner. The fir was backed by 4 inch I-beams. Channels were used for the other boundaries.

B. Basic Procedure for Preparing a Test

Basically the test procedure consisted of compacting soil in a model cell, then as air pressure was applied in increments, various measurements were made on the system. A typical procedure is here described for the preparation of a test on loess using the smaller model cell.

The loess was sieved through a number 20 sieve to break up clods and to eliminate all plus 20 particles including lead shot. The model cell was placed on end as shown in Figure 6 and a rubber boot was laid over the innertube in the bottom of the cell. The boot had been cut from 1/16 inch gum sheet rubber to fit the inside of the cell. The loess was then placed in the cell in layers and compacted. Each layer weighed 970 grams (about one inch thick after light compaction). Before each layer was compacted it was leveled by means of a screed. Compaction was accomplished by lowering a block of wood onto the soil surface and by dropping a Standard Proctor hammer a given number of times for a given height of fall with the blows arranged in a pattern as shown on top of the block in Figure 7. When 12 blows per layer were required, this pattern was duplicated in reverse for each layer. Also each alternate layer was compacted by a reversed pattern of blows. On the bottom of the block, finish nails were placed with just the heads protruding out of the wood. The finish nails had been previously ground to an approximately spherical shape. The impressions of these nail heads on the soil provided seats for placement of lead shot for the X-ray tests. The compaction blocks shown for clay and loess were so near the dimensions of the cell that they bound tightly against the walls of the cell when sand was compacted. It became necessary to cut a

Figure 6. Small model cell in position for compacting soil in place

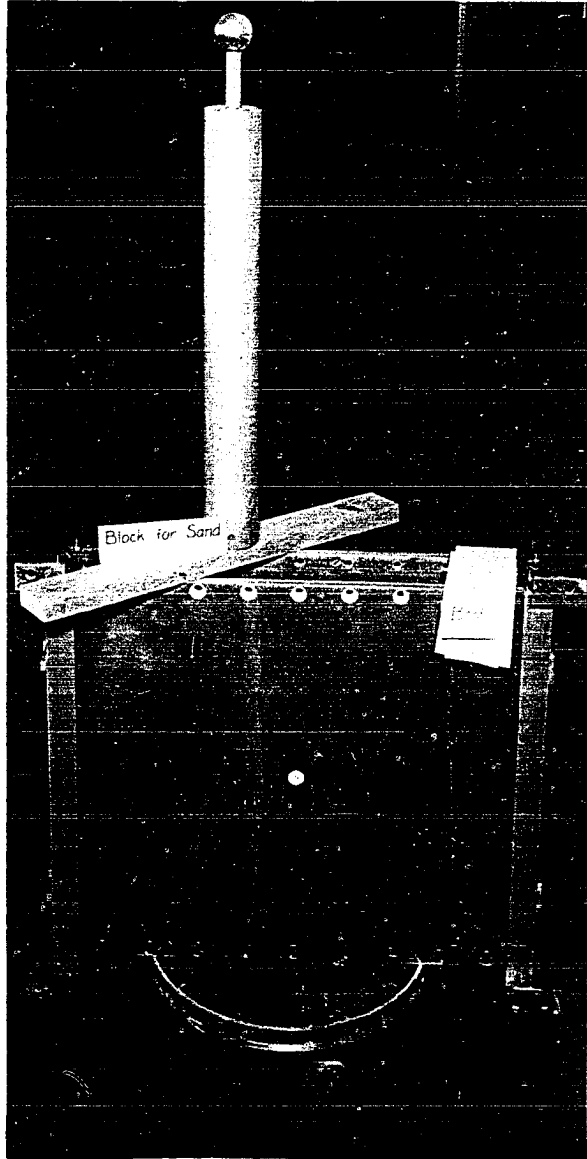
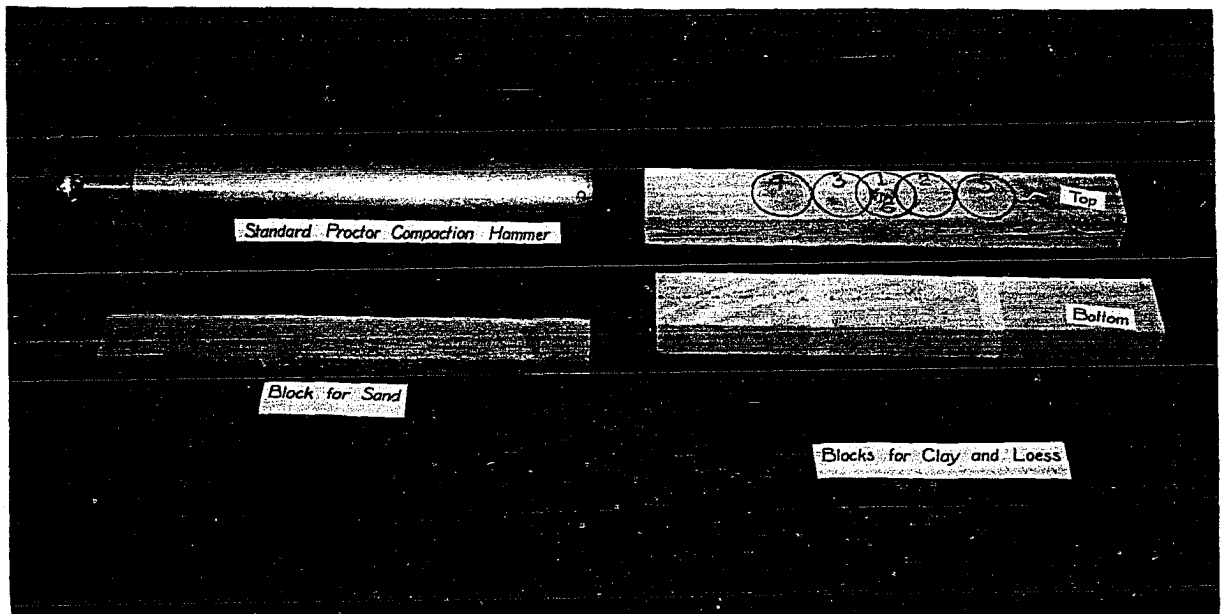


Figure 7. Blocks on which Standard Proctor Hammer was dropped for soil compaction showing the pattern of blows on the top and the finish nail heads for shot spacing on the bottom of the blocks for clay and loess



smaller block out of 1 inch hardwood to use with sand. See Figures 6 and 7. The total weight of soil required to fill the cell was measured, the top surface was accurately screeded to finished height and the top bar (with innertube attached) was securely screwed into place. At this point the cell was ready for control tests which did not require placement of the flexible pipe model. Such tests included the control X-ray test for displacement of shot in the soil.

C. Procedure for Obtaining Load-Deflection Data

The cell, packed as described above was carefully tipped over into the position shown in Figure 8 such that the z-axis was vertical. In order to obtain load-deflection data, a model pipe section had to be installed. The top aluminum plate was removed. The model pipe section was positioned on the top of the soil, as shown in Figure 8, then by alternately forcing the pipe model into the soil a fraction of an inch and by scooping out the soil inside, the pipe model was lowered into position as shown in Figure 9. The dial gage is seen in its proper position in this same photograph. In order to allow a floating action so that the gage might follow the pipe model during deflection, the gage was mounted on polished steel balls in rings as shown in the cutaway section of Figure 10. Most of the pipe models were cut from

Figure 8. Small model cell packed with loess in position for testing with the Z-axis vertical and with the model flexible pipe section positioned for lowering into place



Figure 9. Small model cell with model pipe section in place and with the dial gage in proper position for measuring pipe deflection

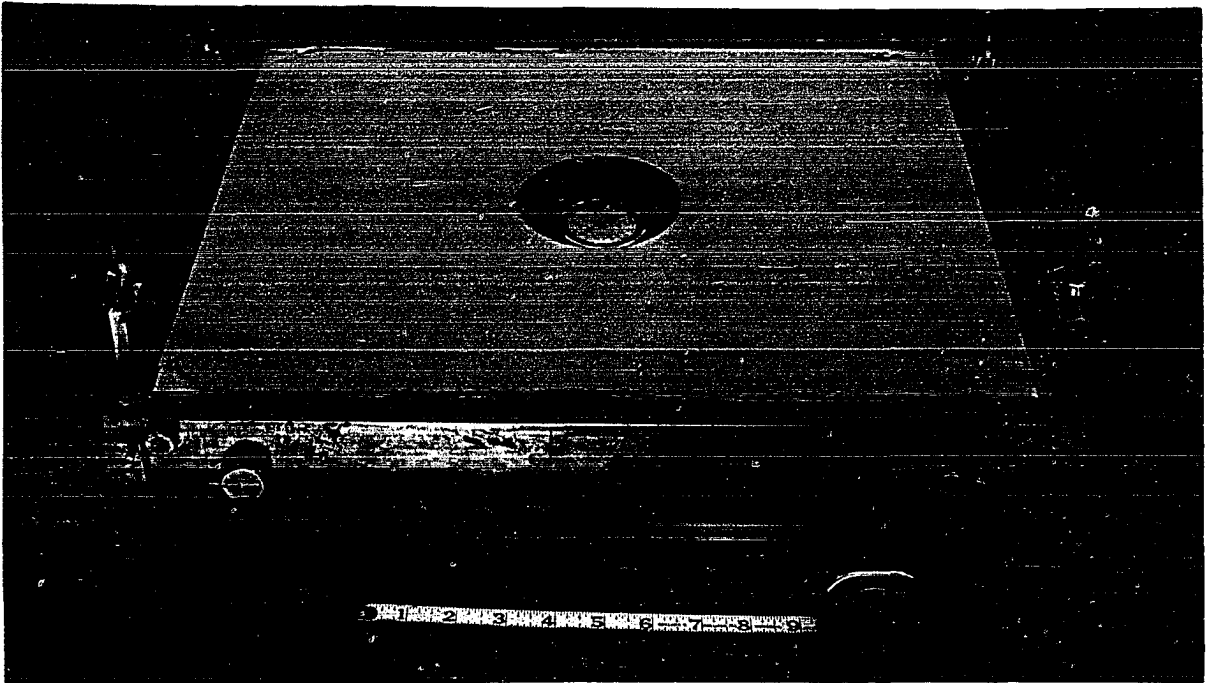
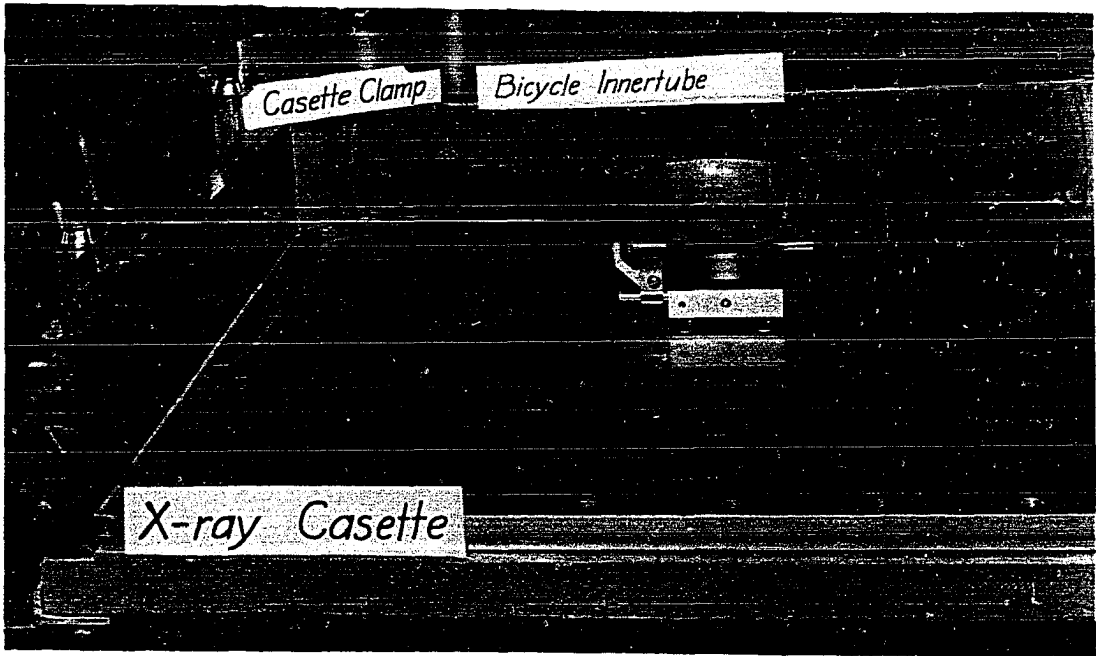


Figure 10. Cutaway section of model pipe in small model cell showing dial gage mounted on polished steel balls for following pipe displacement and showing how the X-ray cassette is raised into position by the cassette clamps



tin cans, but to vary the EI of the pipe models, heavier sections were also made up from galvanized sheet iron. Since the model flexible pipe sections all had a longitudinal joint or seam in them, the seam was always placed at a position of about 45° of arc from the principle diameters of the deflected model pipe. This position is approximately the point of counterflexure of the pipe wall, so the variation of pipe wall stiffness at the seam should have a minimum influence at this position.

With the model pipe and dial gage in place the top aluminum plate was securely replaced, the observation hole was unplugged, and the cell was ready for applying pressure. Pressure was provided by a bottle of oxygen connected through appropriate tubing as shown in Figure 11. The pressure control on the bottle made it possible to hold a given pressure in the innertube regardless of any leakage. As increments of pressure were applied, the dial gage was read through the observation hole. Pressure increments were usually 5 p.s.i. and the time rate at which these readings were taken was held as nearly constant as could be estimated. This was done to reduce the effect of a slight time lag in the dial gage reading after each pressure increment was applied.

Figure 11. Load-deflection test in process on the small model

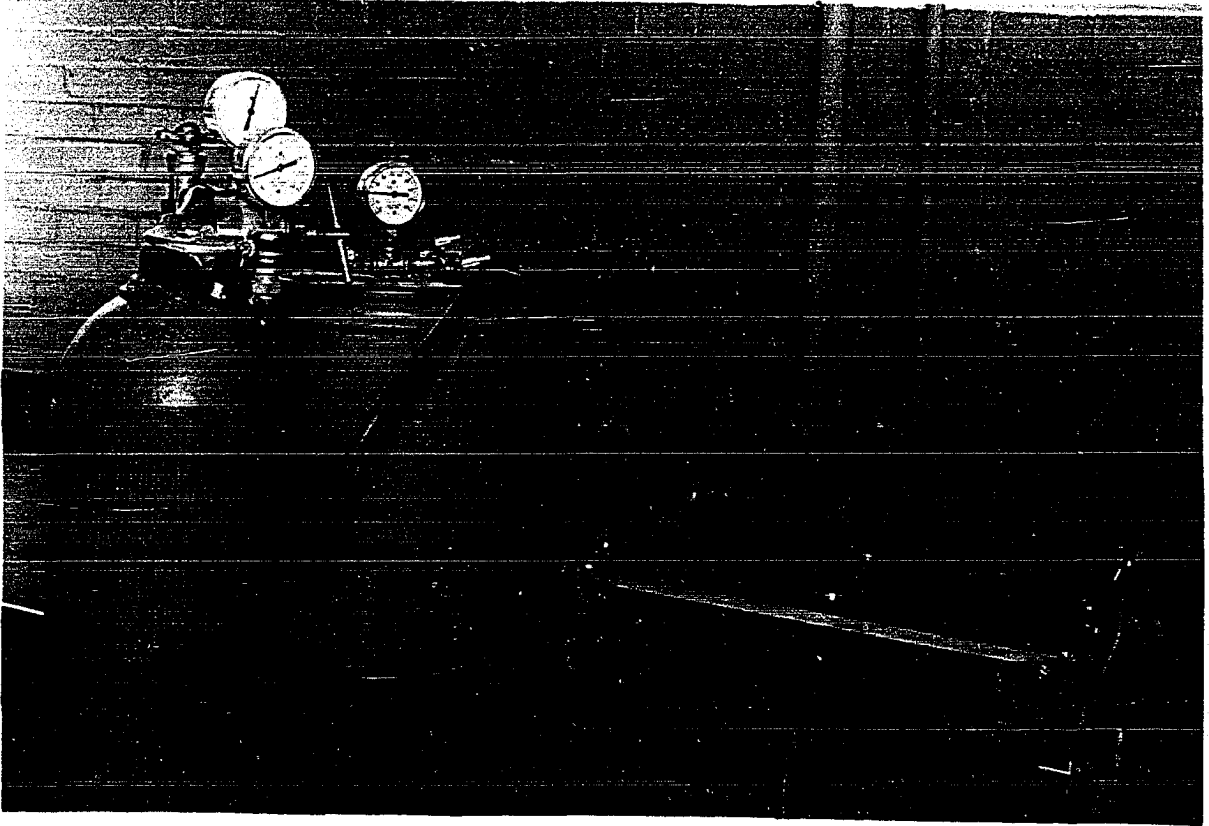


Figure 12. Small model cell assembled and ready for attaching the air pressure leads for measuring reaction pressure at the center of the cell

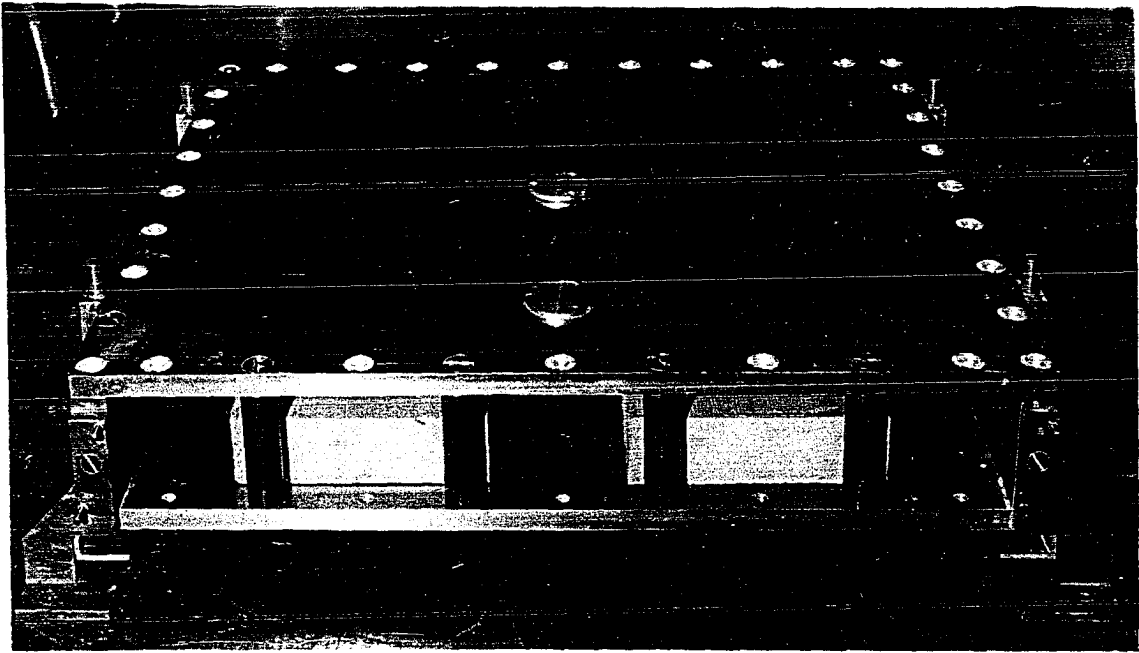
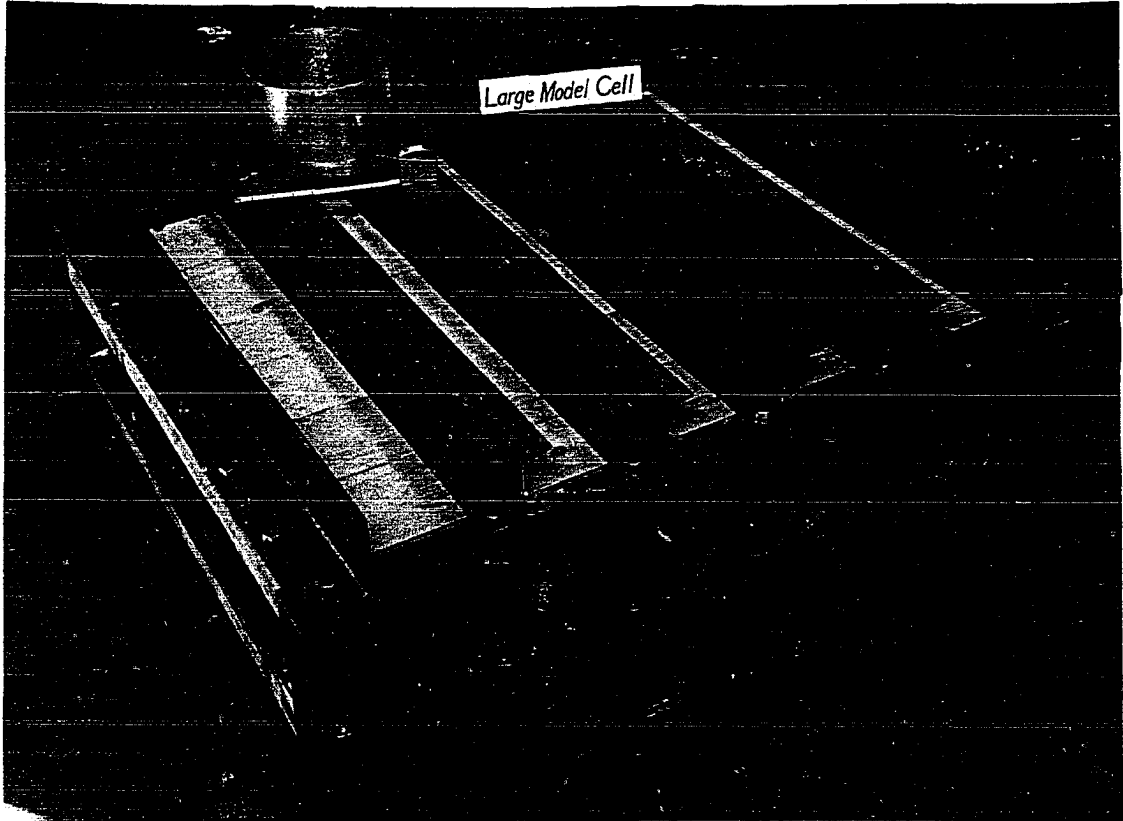


Figure 13. Large model cell and model pipe section so designed that all linear dimensions in the confined soil are twice as great as in the small model



D. Procedure for Making X-ray Photographs

When the cell was prepared for x-ray testing it was prepared just as described above except that the dial gage was omitted. Instead a row of shot was taped around the inside perimeter of the pipe section to insure that the deformation of the pipe might be carefully defined on the film. Lead shot was also placed at about 1 inch intervals on a grid throughout the soil. The X-ray cassette could be drawn up under the cell by means of the cassette clamps as shown in Figure 10. A separate X-ray was made after each increment of pressure. By superimposing the X-ray photographs the soil displacement could be discerned by means of the shot pattern.

E. Procedure for Determining Net Soil Pressures at Various Positions in the Cell

One series of tests called for a determination of soil pressure at various points in the cell when a given pressure was applied in the innertubes. This was accomplished by compacting soil in place with the cell on its end just as described under Basic Procedure for Preparing a Test, but the soil level was raised only to a given point in the cell. The top bar (with innertube attached) was then lowered into position just over the top of the soil but with sufficient clearance to allow some inflation of the innertube. The top

bar was then held in place with blocks and about 1.8 p.s.i. pressure was introduced to hold the soil in place while the cell was lowered from its end to the position shown in Figure 12. Pressure increments were applied through the other innertube. These pressures were called the load pressures. The resulting pressures on the innertube within the cell were measured and designated as reaction pressures. This test made it possible to determine how much of the load pressure actually reached various points in the cell.

VI. RESULTS OF INVESTIGATION

A. Influence of Height of Fill on Modulus of Soil Reaction

The influence of height of fill on the modulus of soil reaction, e_r , is the first result available from any given load-deflection test. As described on page 40, e cannot be conveniently determined from its definition since it is difficult to measure the quantities involved. A better approach is to resolve the Iowa Formula in terms of e_r and then by means of a model to measure the necessary quantities for the evaluation of e_r . Of course e_r for the model is the same as e_r for the prototype. This method of evaluation has the advantage that e_r is correct for use in the Iowa Formula to predict deflection.

The Iowa Formula is rewritten here for convenience:

$$\Delta x = \frac{K W_c r^3}{EI + 0.061(e_r)r^3} \cdot$$

Resolving this equation for e_r ,

$$e_r = 1.36 \frac{W_c}{\Delta x} - 16.4 \frac{EI}{r^3} \quad \text{Eq. 12}$$

where K is assumed to be 0.083. At this point it is proposed to rewrite Equation 12 in a form involving height of fill, H ,

and then to compare it with the results of actual model tests in order to determine the influence of H on e_r . The second term on the right is a constant for any given pipe and represents the effect of pipe stiffness on the modulus of soil reaction. It is considered in detail under "Influence of Pipe Wall Stiffness on the Modulus of Soil Reaction".

The first term on the right is a constant times $\frac{W_c}{\Delta x}$.

x can be measured in the model pipe, but W_c must be rewritten in terms of the height of fill which is related by a constant coefficient to the load pressure in the model cell. The load pressure is simply the air pressure in the inner-tubes minus a small correction pressure for tare inflation of the innertubes. W_c may be evaluated according to Equation 2 which is rewritten here.

$$W_c = C \gamma D^2 \quad \text{Eq. 2}$$

where W_c is the load on the pipe per unit length; D is the diameter of the pipe; γ is the unit weight of soil; and C is a coefficient dependent on the internal friction angle of soil, culvert condition (complete or incomplete, ditch or projection), the ratio $\frac{H}{D}$, where H is height of fill, and settlement and projection ratios. Spangler (16, pp. 424, 426) has plotted values of C as a function of $\frac{H}{D}$ for various culvert conditions. See the reproduction of Spangler's plots on Figure 14. (He has demonstrated that the internal friction angle of soil has a negligible effect on C for practical

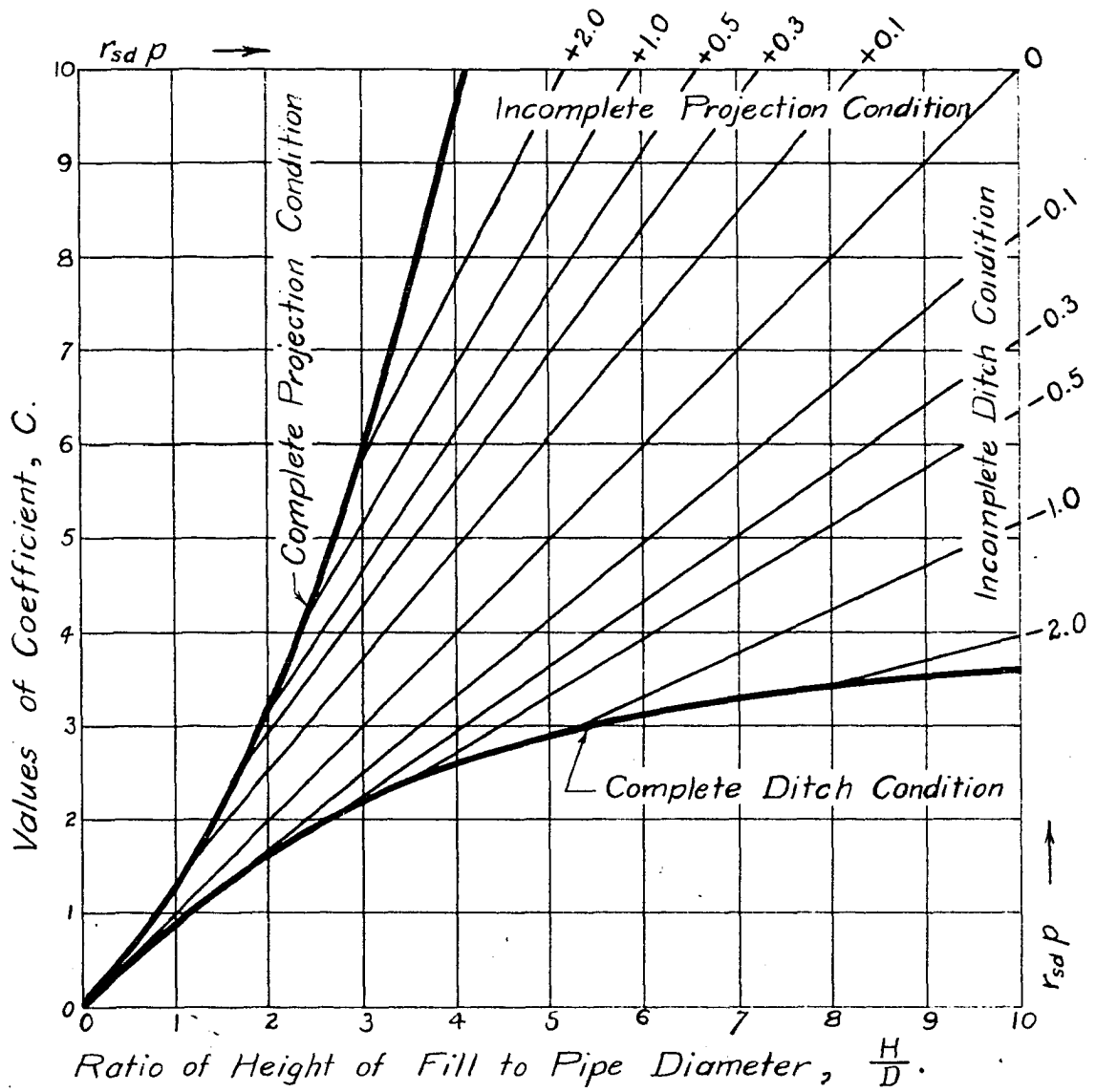


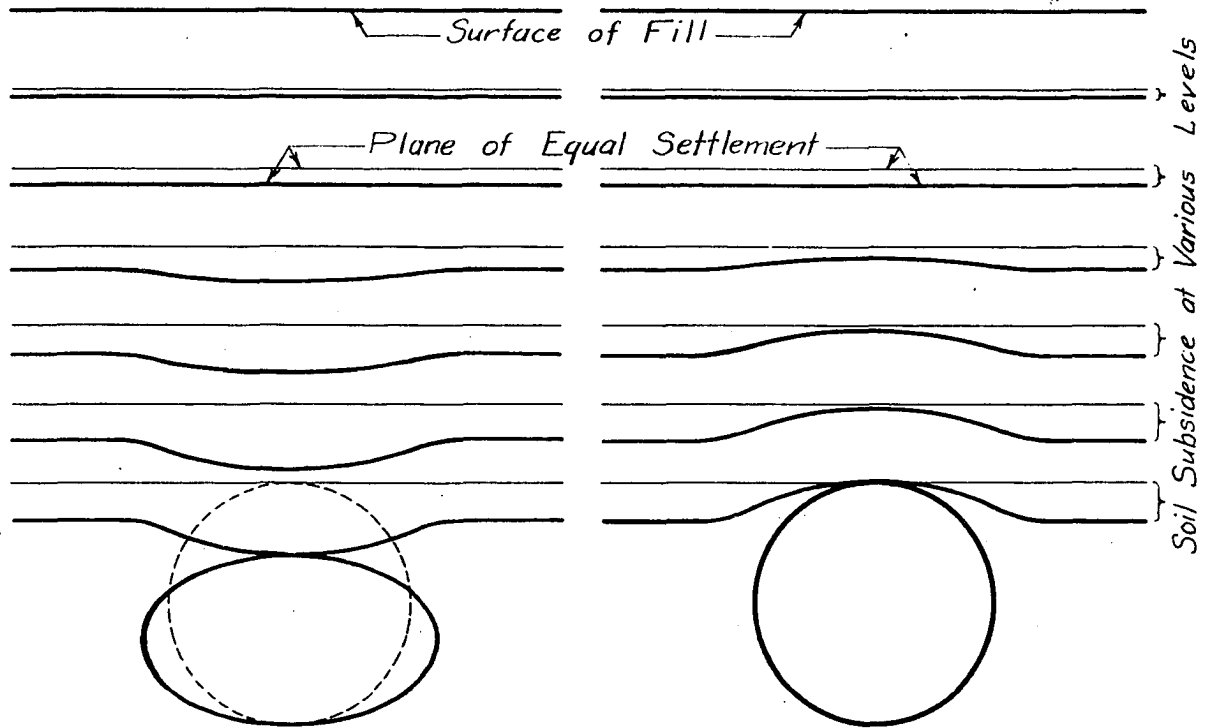
Figure 14. Graphical solution of coefficient, C , for positive projecting conduits according to Spangler (16, p. 42)

purposes.) His plots indicate a straight line variation between $\frac{H}{D}$ and C , but it should be pointed out that they do not all pass through the origin and that only for a settlement ratio and/or projection ratio equal to zero does the plot extend down to the origin. Since this investigation is limited to high fills, there is no concern about the lower portion of the plots, and an equation for C may be written as follows:

$$C = C_1 \frac{H}{D} + C_2 . \quad \text{Eq. 13}$$

In the case of high fills C_2 can be set equal to zero as the following argument justifies.

Since culverts, unlike most pipes, are not designed to withstand high internal pressure, they can be relatively flexible. Generally the magnitude of decrease in vertical diameter is comparable to the vertical compression of the adjacent soil of equal height. The stiffness of the pipe and compressibility of the soil determine whether the pipe decreases in height more than or less than the adjacent soil, but for an economically designed flexible pipe it is very improbable that decrease in vertical diameter is less than the decrease in height of the adjacent soil. Such a condition is shown in Figure 15a and is defined by Marston (8) as the incomplete ditch condition. A possible exception to the incomplete ditch condition would occur if the soil were placed around the culvert in such a loose state that the vertical compression of the soil adjacent to the pipe exceeded



(a) Incomplete ditch condition

(b) Incomplete projection condition

Figure 15. Two basic settlement conditions for pipe-fill systems

5 per cent. Of course, 5 per cent decrease in vertical (or 5 per cent increase in horizontal) diameter of the pipe is considered design limit. In this case the pipe would need to be stiff enough to keep deflection within 5 per cent. Such a relationship between pipe and soil is defined by Marston as the incomplete projection condition. See Figure 15b. For rigid pipe such as concrete or cast iron the incomplete projection condition is important, but it may be ignored in essentially all flexible pipe considerations as it is more economical to compact the fill sufficiently to reduce compression below 5 per cent. Spangler's plots of C as a function of $\frac{H}{D}$ for ditch condition of conduits are based on the product of projection ratio and settlement ratio. He defines projection ratio, p , as the ratio between the vertical distance from the top of the pipe to the natural ground surface and the diameter of the pipe (16, pp. 418, 425). The next paragraph arrives at a reasonable value for p .

It was demonstrated on page 57 that for flexible culverts under high fills it is conservatively acceptable to assume that relative deflection of pipe with respect to soil is symmetrical about the horizontal diameter of the pipe. With this assumption of vertical symmetry, the equivalent natural ground surface is at the level of the horizontal pipe diameter. The projection ratio, p , is then 0.5.

Settlement ratio, r_{sd} , is defined as the ratio of the

difference in settlement between the top of the pipe and the adjacent soil which was originally at the level of the top of the pipe and the vertical compression of an adjacent column of soil of height initially equal to the pipe diameter. For an economical, flexible pipe the settlement ratio will be very small and it will be negative; since the top of the pipe settles slightly more than the adjacent soil at the same level. Typical values for r_{sd} are calculated from the X-ray photographs. From measurements made on the X-ray photograph of a model pipe of 0.011 inch thick steel in loess the settlement ratio is -0.6 at 80 psi. inertube pressure. See Figures 34 and 36 for vertical deflection of pipe and compression of soil respectively. For a heavier model pipe of 0.019 inch steel, the settlement ratio was found to be about -0.4 at 80 psi. For a model pipe of 0.034 inch steel the settlement ratio was about -0.1. As the stiffness of the model pipe continued to increase, the settlement ratio would become positive and the incomplete projection condition would obtain. The lightest model pipe exceeded 5 per cent deflection at an equivalent height of fill of about 16 feet which might be considered near the lower limit for flexible pipe culverts under high fills, so the maximum numerical value for r_{sd} might be reasonably set at -0.6.

The resulting product of projection ratio and settlement ratio is

$$r_{sd}^p = -0.3.$$

Now for this particular value, Spangler's plot of C versus $\frac{H}{D}$ intersects the C axis at $C_2 = 0.2$. The equation for C becomes

$$C = 0.690 \frac{H}{D} + 0.2.$$

For high fills the constant, $C_2 = 0.2$, can be ignored without affecting the accuracy commonly accepted in culvert design.

Even if Spangler's plot for $r_{sd}^p = -1.0$ were assumed, the equation for C becomes

$$C = 0.463 \frac{H}{D} - 0.5.$$

Under this improbable situation, if the height of fill were 10 diameters, the error incurred by ignoring $C_2 = -0.5$ is only about 11 per cent. So for all practicability, Equation 13 may be written in the form:

$$C = C_1 \frac{H}{D}$$

and therefore, $W_c = C_1 \gamma H D$ where C_1 is a constant for a given pipe-fill system. Substituting the above relationship in Equation 12 the modulus of soil reaction as a function of height of fill according to Spangler's theory becomes:

$$e_r = 1.36 \frac{C_1 \gamma H D}{\Delta x} - 131.2 \frac{EI}{D^3}. \quad \text{Eq. 14}$$

It is more convenient to compare this theoretical relationship with the experimental test results if Equation 14 is rewritten in terms of H as a function of Δx , i.e.,

$$H = \frac{(e_r + 131.2 \frac{EI}{D^3})}{1.36 C_1 \gamma D} \Delta x. \quad \text{Eq. 15}$$

An equivalent empirical relationship may be developed from the experimental load-deflection diagrams. See Figures 16 to 18. In these diagrams the ordinate, P, is the inner-tube pressure and the abscissa, Δx , is the increase in horizontal diameter of the model pipe section. Since P is a pressure rather than a height of fill as called for in Equation 15, it is necessary to convert P to H by means of the relationship $TP = H\gamma$, where γ is the combined unit weight of the soil and the coefficient, T, is a load transmission ratio which is constant for a given pipe-fill model system. It is discussed and evaluated under "Effect of Friction of the Cell Walls on the Vertical Soil Pressure." Actually there should be another correction term included in the conversion of P to H. Figures 22, 23, and 24 show a tendency for all load-deflection diagrams to converge at a value of P of about 5 psi. or a little more. This correction represents the tare pressure necessary to inflate the inner-tube and should be subtracted from P when converting to H. Since this correction represents only 1 to 2 feet of average earth fill it could be neglected, but in the calculations of this section it is subtracted from P.

A cursory inspection of all load-deflection diagrams reveals that after initial soil adjustments have taken place

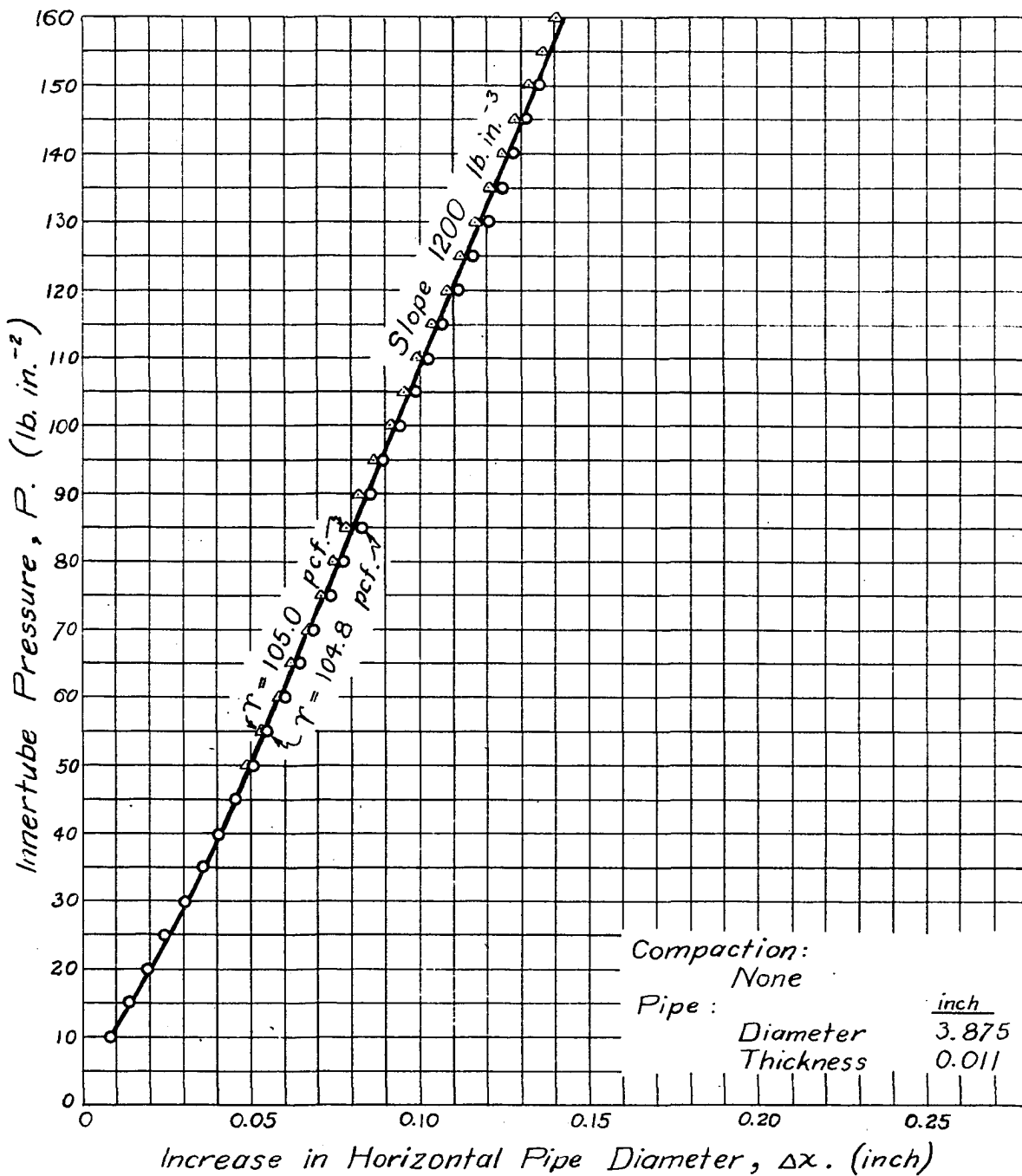


Figure 16. Load-deflection diagrams for duplicate tests on white silica sand (Saint Peter formation) in small cell

the diagrams are essentially linear up to a horizontal deflection of about 0.27 inch or 7 per cent increase in horizontal diameter. Above 7 per cent the slopes tend to change. This may be due in part to the fact that the horizontal diameter is approaching its maximum.

It is interesting to note that the model pipe sections generally collapsed between 9 per cent and 13 per cent horizontal deflection. It is suspected that these values may be slightly lower than in field installations since the load pressure was not always applied outside of the plane of equal settlement. Fortunately the design limits of 5 per cent make it unnecessary in this project to consider the load-deflection diagrams beyond the first linear portions.

An empirical equation for the linear portion of the plots would be

$$P = S \Delta x + P_0 \quad \text{Eq. 16}$$

where S is the slope and P_0 is the P -intercept. S and P_0 are constants for any given pipe-fill system. Converting P to H , Equation 16 becomes

$$H = \frac{TS}{\gamma} \Delta x + \frac{TP_0}{\gamma} \quad \text{Eq. 17}$$

Now if theoretical Equation 15 is to apply to experimental Equation 17 the coefficients of Δx must be equal, or

$$\frac{(er + 131.2 \frac{EI}{D^3})}{1.36 G_1 \gamma D} = \frac{TS}{\gamma} \quad \text{Eq. 18}$$

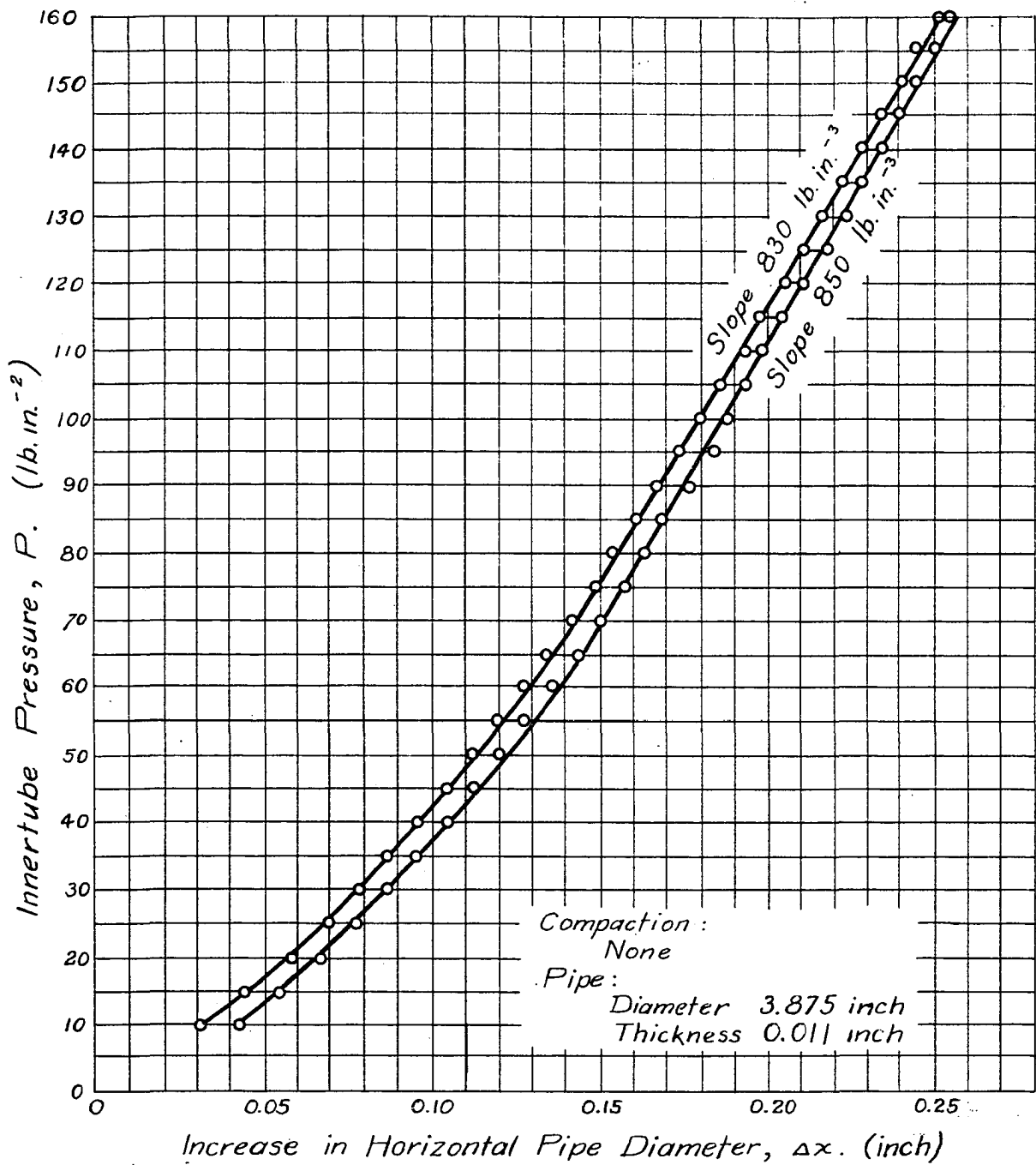


Figure 17. Load-deflection diagrams for duplicate tests on moist (bulked) plaster sand in small cell

This equation establishes the condition that e_r must be a constant for a given pipe-fill system since all other quantities are constant. In other words e_r is independent of height of fill. This result is significant.

A visual check is found on the superimposed X-ray data of Figures 35 and 34. At the soil-innertube interface the soil displacement follows closely the typical compression diagram for soils (23, p. 218) wherein the settlement varies linearly as the logarithm of the load pressure. It must be noted, however, that these figures do not show the same variation in soil displacement adjacent to the pipe. Rather the ratio of load to displacement appears constant. This remarkable relationship confirms the conclusions that e_r is independent of height of fill, H .

Now to complete the application of theoretical Equation 15 to experimental Equation 17, P_0 must be zero. Unfortunately the load-deflection diagrams show that this is not the case. For the great majority of installations, however, it may be acceptable to assume that H_{p_0} is zero. Such an assumption is not without justification as described in the following paragraphs.

In the case of low degree of compaction the value of P_0 is negative. See Figures 22 to 24 inclusive. Neglect of P_0 in such a case would result in a calculated deflection less than the actual deflection for a given height of fill so the

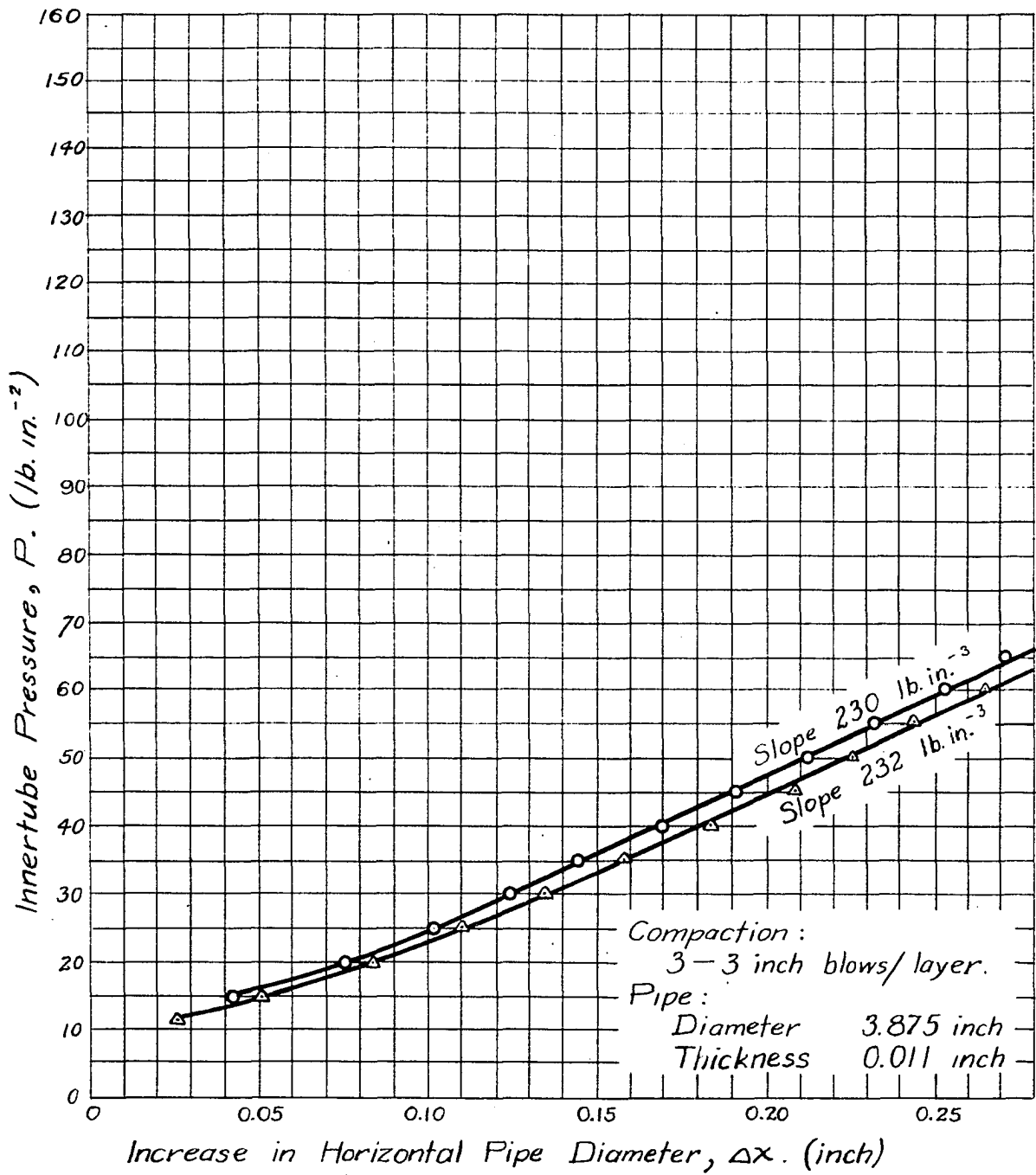


Figure 18. Load-deflection diagrams for duplicate tests on loess in small model cell

error is on the unsafe side. In view of this fact it would seem advisable to reject as unsatisfactory any installation in which the degree of compaction is so low. As the degree of compaction increases the intercept, P_0 , becomes positive so its neglect would lead to conservative results; that is, the actual deflection would be less than the predicted deflection. The amount of the error involved varies according to the per cent deflection allowed in the design of the pipe. If 5 per cent deflection of the pipe is allowable, and if compaction is specified to be greater than that shown at the point designated as critical compaction, tests on Saint Peter sand show that about a 3 per cent error is involved. See Figures 19 and 20. Clearly such an error on the conservative side could be disregarded.

The error of neglecting P_0 is greater in the case of loess and clay. See Figure 21. This figure shows the actual per cent error if a pipe deflection of 5 per cent is assumed for both clay and loess as compared with sand. Compaction is unsatisfactory if P_0 is negative, so this consideration will start with the point designated as critical compaction. If critical compaction is specified there is no error in neglecting P_0 . Indeed as pointed out later, there is no P_0 at critical void ratio. See "Influence of Compaction on the Modulus of Soil Reaction". As the compaction is increased above critical compaction, the error of neglecting

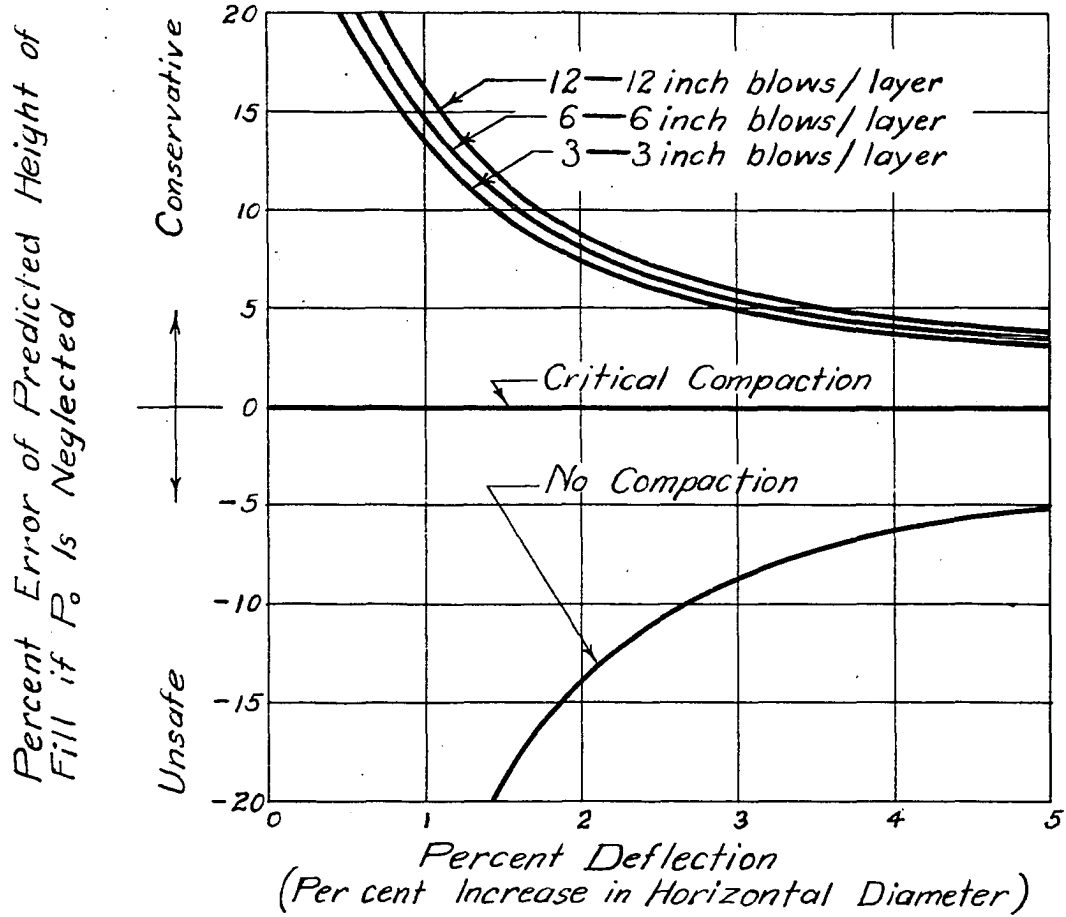


Figure 19. Per cent error of predicted height of fill at various deflections of flexible pipe in white silica sand (Saint Peter Formation) if P_0 is neglected (based on data from Figure 22)

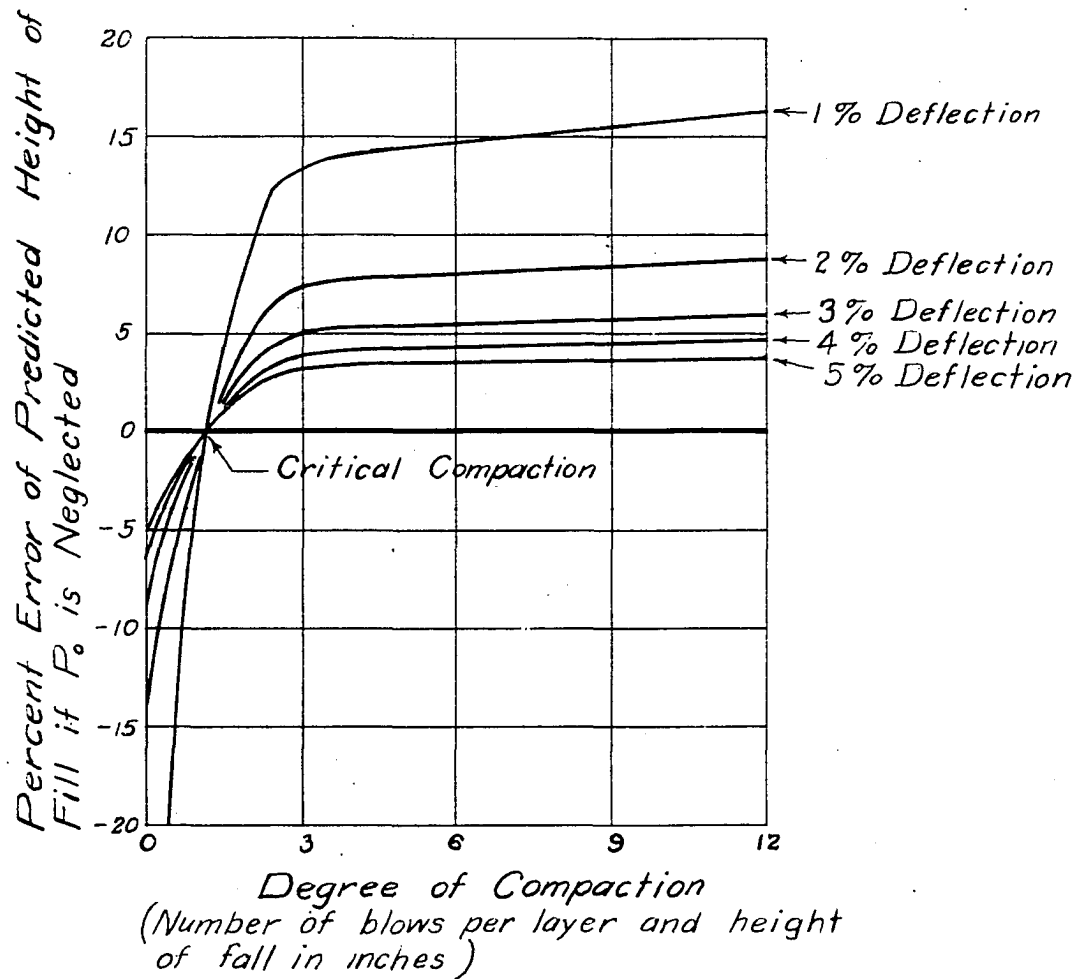


Figure 20. Per cent error of predicted height of fill at various deflections of flexible pipe in white silica sand (Saint Peter Formation) as a function of degree of compaction if P_0 is neglected

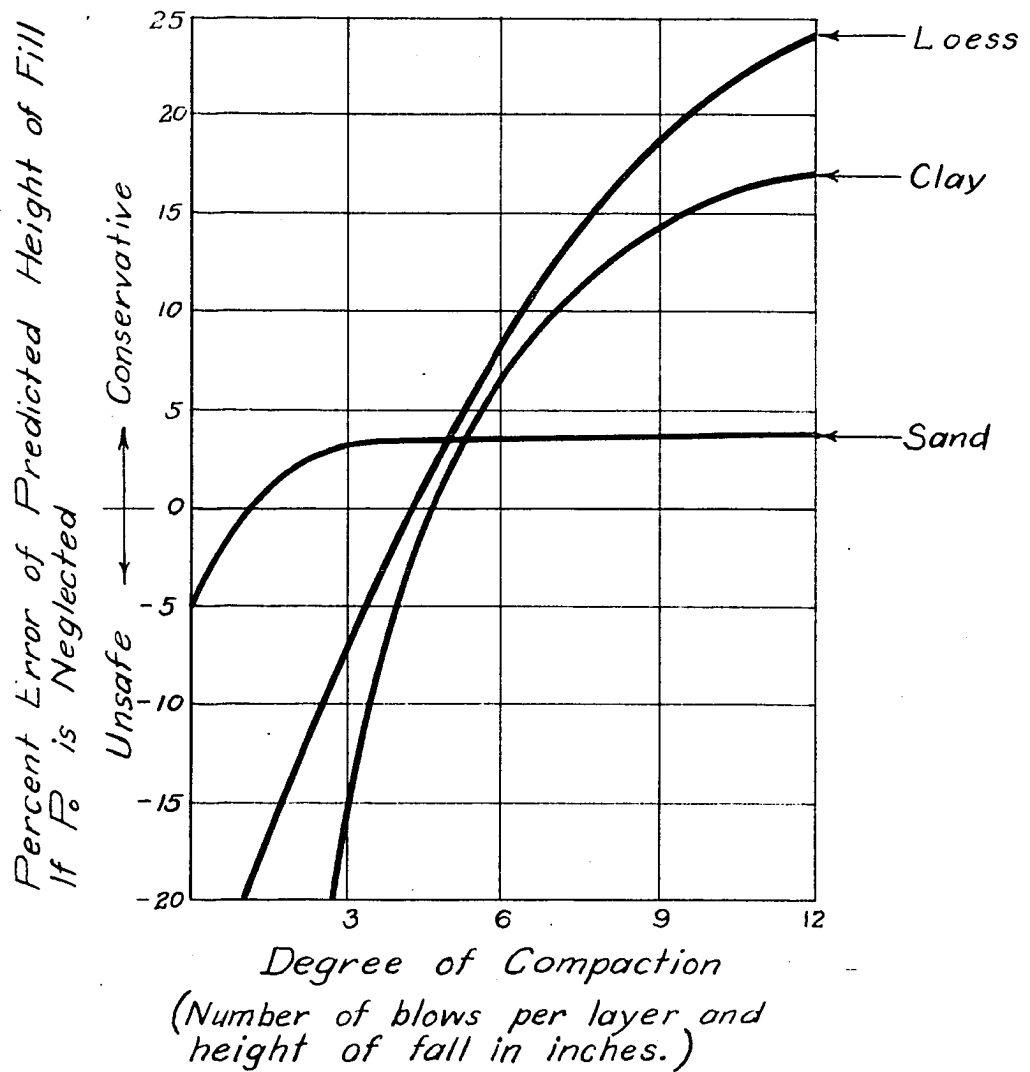


Figure 21. Per cent error of predicted height of fill at 5 per cent deflection of flexible pipe in various soil types if P_0 is neglected (see Appendix F for soil identification)

P_0 , although conservative, becomes rather large. Note, for example, that for loess compacted at twelve 12-inch blows per layer the error is about 24 per cent. On major projects it may prove economical to carefully control the compaction and take advantage of the 24 per cent which is otherwise lost.

One other argument may be presented in favor of neglecting P_0 . The exact value of the intercept appears to be somewhat sensitive. For instance three plots of load-deflection tests under supposedly identical circumstances are shown in Figure 27. Test 1 was performed at an earlier date than the other two. Though the straight-line portions of the plots are nearly parallel, Test 1 is vertically displaced from the other two by about $P_0 = 18$ psi. There is indication that the initial conditions of soil-pipe bearing are different in the case of the one. Such differences will very likely show up in actual field installations as well. For example if equipment is moved over the pipe while the fill is very little higher than the top of the pipe, an initial bearing condition might be developed between the pipe and the fill which would cause a deviation in the intercept of the load-deflection diagram. Conservative design methods would certainly safeguard against such a possibility by neglecting the intercept correction. Nevertheless, if sufficient control of the installation were possible, economy may require that

the intercept correction, P_0 , be included in the design. Under these circumstances Spangler's deflection formula would need modification by a small additive constant. A proposed modification is found under "Modification of the Spangler Theory". It should be reemphasized that the need to consider P_0 would occur very rarely.

B. Influence of Compaction on the Modulus of Soil Reaction

Figures 22, 23, and 24, for sand, loess, and clay respectively show how the load-deflection curves vary if all conditions are identical except degree of compaction of the soil. In every case there appears to be a very definite increase in the slope, S , and also an increase in the P -intercept, P_0 , as the degree of compaction increases.

The increased slope is easily accounted for since it is based on pipe deflection, Δx , which in turn is a function of the friction angle of the soil inasmuch as friction angle is a measure of the resistance of the soil to relative shear displacement. The friction angle is determined to a large degree by the density or degree of compaction of the soil.

Less obvious is the effect of compaction on P_0 . For low compaction it may be seen that P_0 is negative. As compaction increases, P_0 increases into the positive range. Evidently the phenomenon is related to the critical void ratio phenomenon. Void ratio is defined as the volume of

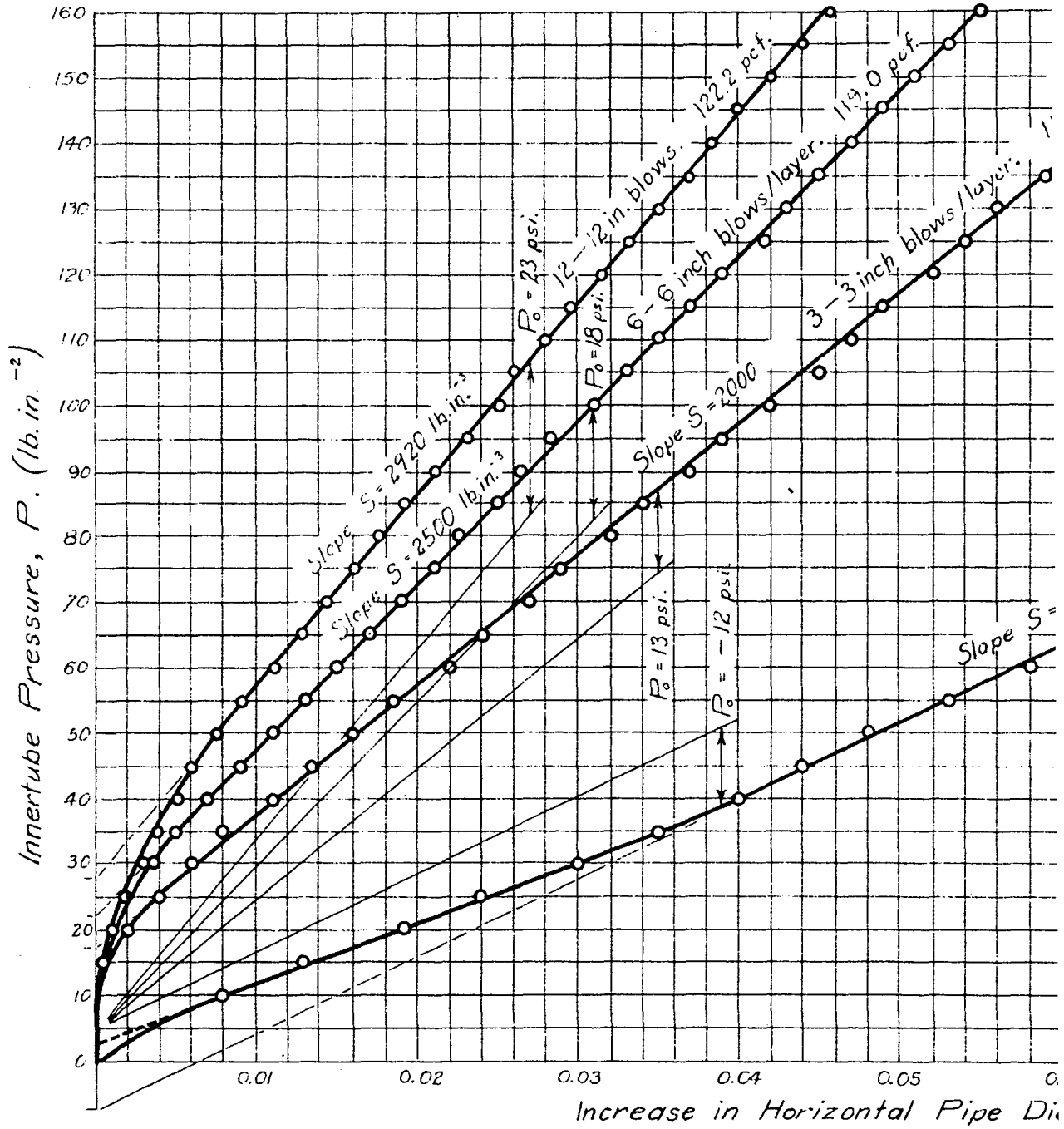


Figure 22. Load-deflection diagrams for white silica sand (Saint Peter Formation) for varying degrees of compaction using small model cell and 3 7/8 inch diameter pipe with 0.011 inch wall thickness

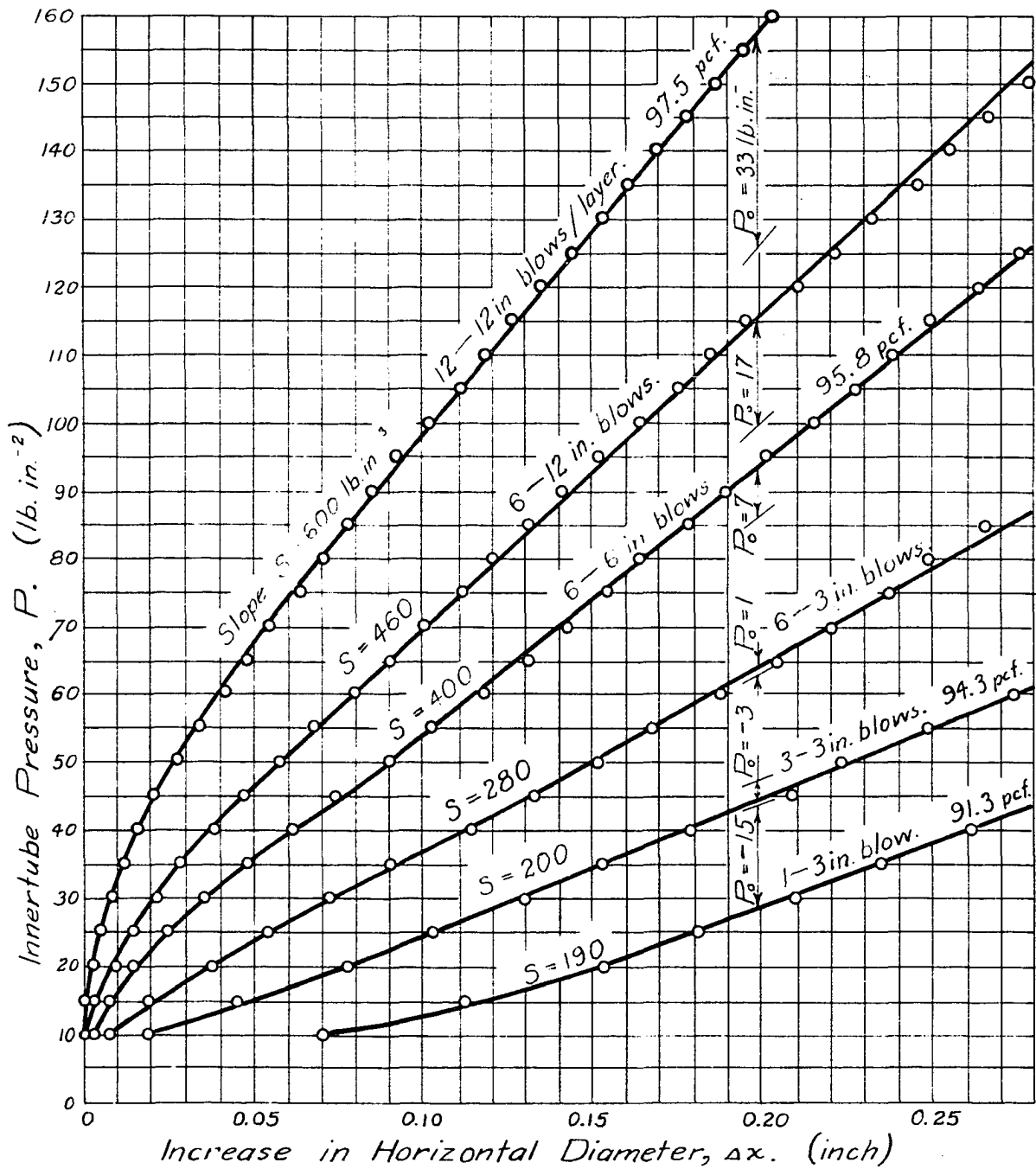


Figure 23. Load-deflection diagrams for loess at varying degrees of compaction using small model cell and 3 7/8 in. diameter pipe with 0.011 in. wall thickness (see Appendix F for soil identification)

P_0 , although conservative, becomes rather large. Note, for example, that for loess compacted at twelve 12-inch blows per layer the error is about 24 per cent. On major projects it may prove economical to carefully control the compaction and take advantage of the 24 per cent which is otherwise lost.

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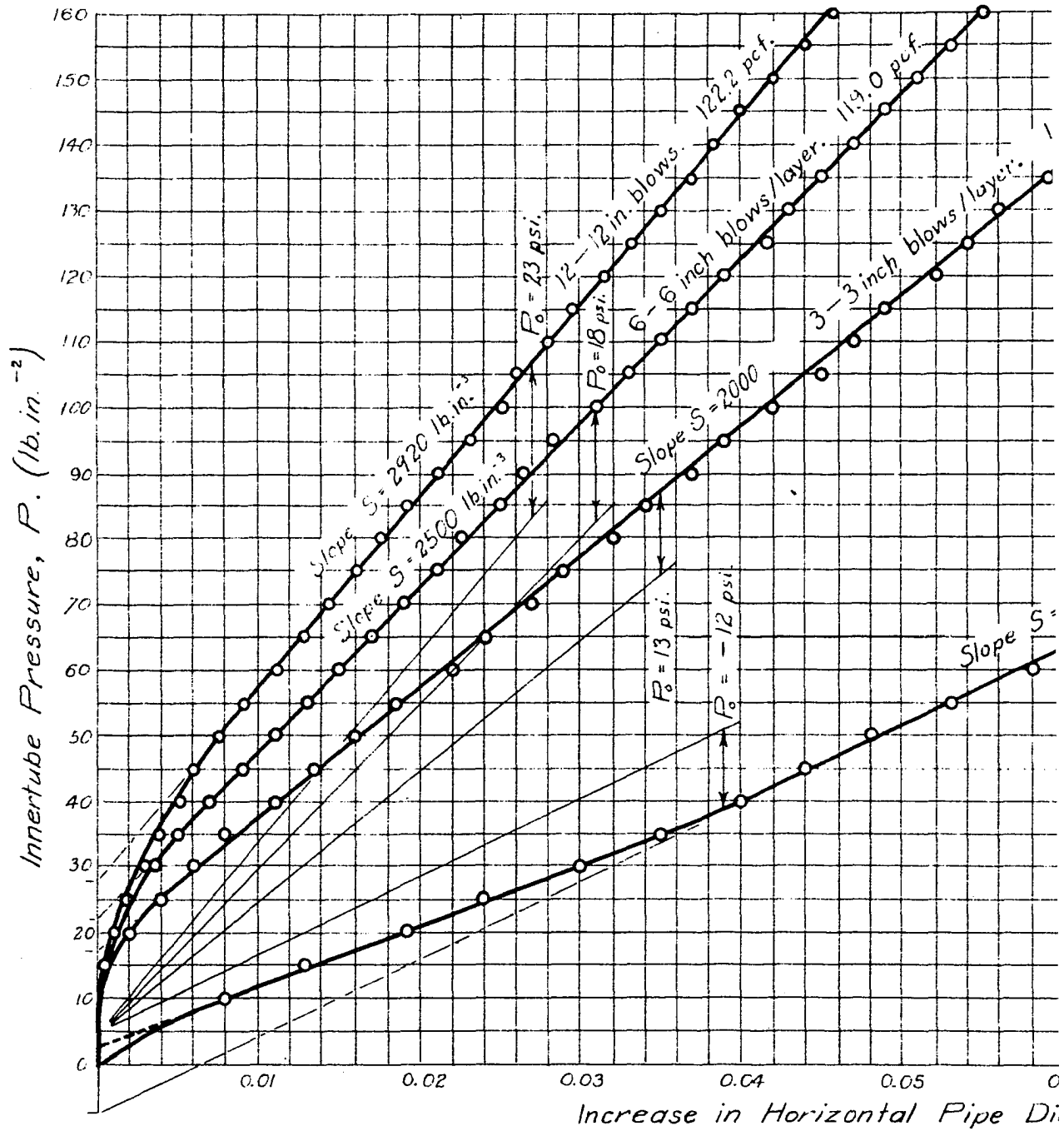
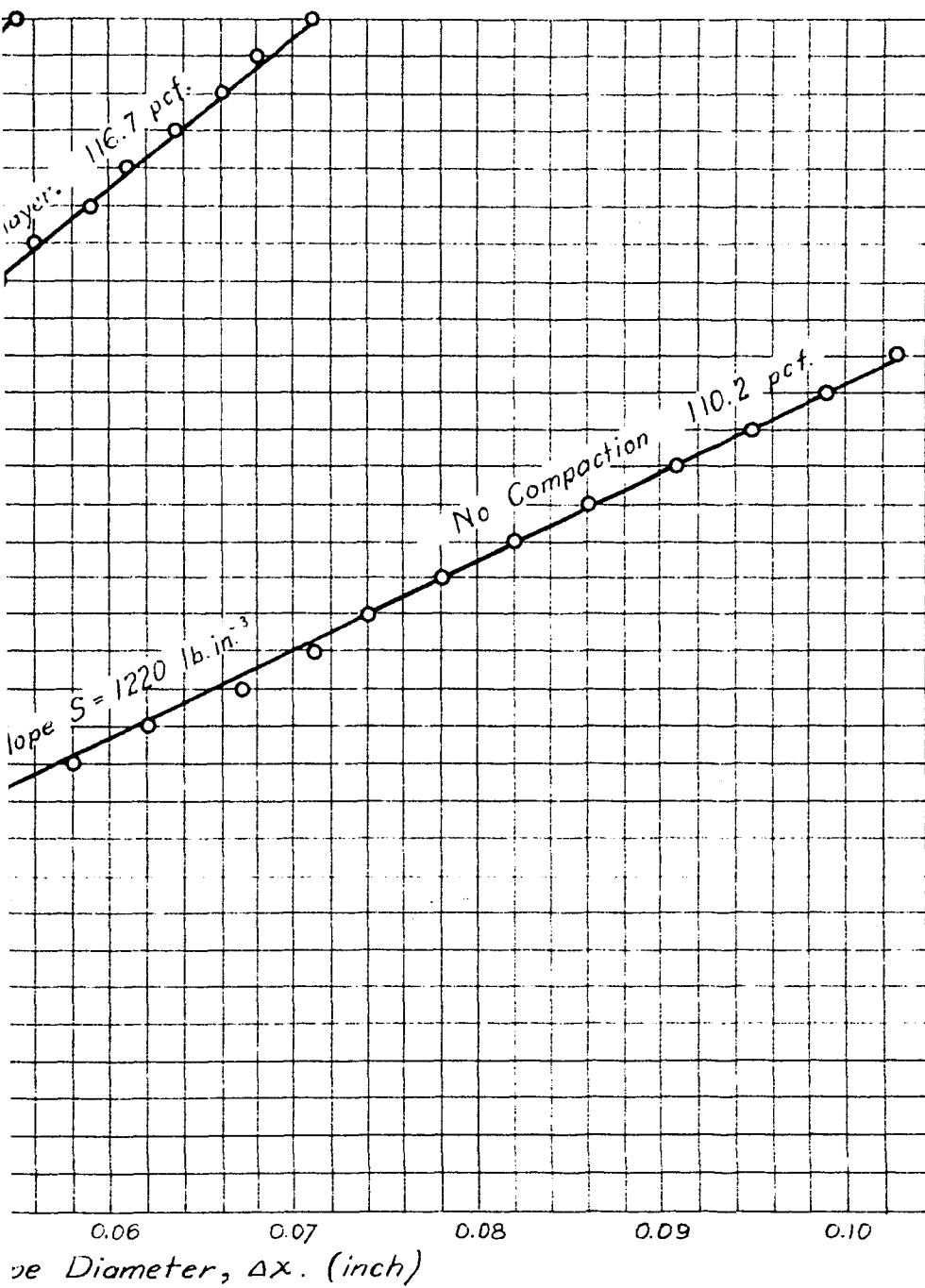


Figure 22. Load-deflection diagrams for white silica sand (Saint Peter Formation) for varying degrees of compaction using small model cell and 3 7/8 inch diameter pipe with 0.011 inch wall thickness



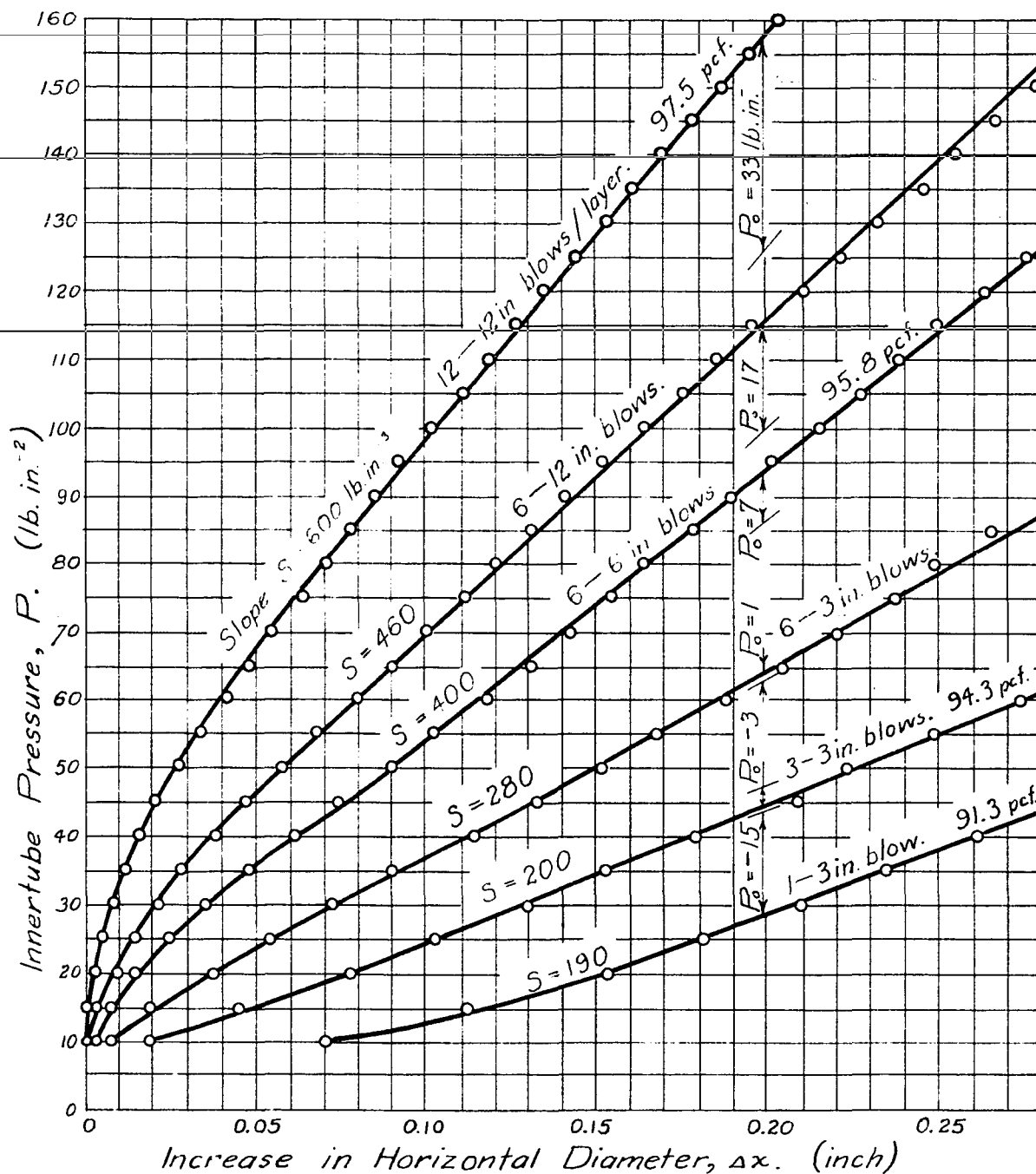


Figure 23. Load-deflection diagrams for loess at varying degrees of compaction using small model cell and 3 7/8 in. diameter pipe with 0.011 in. wall thickness (see Appendix F for soil identification)

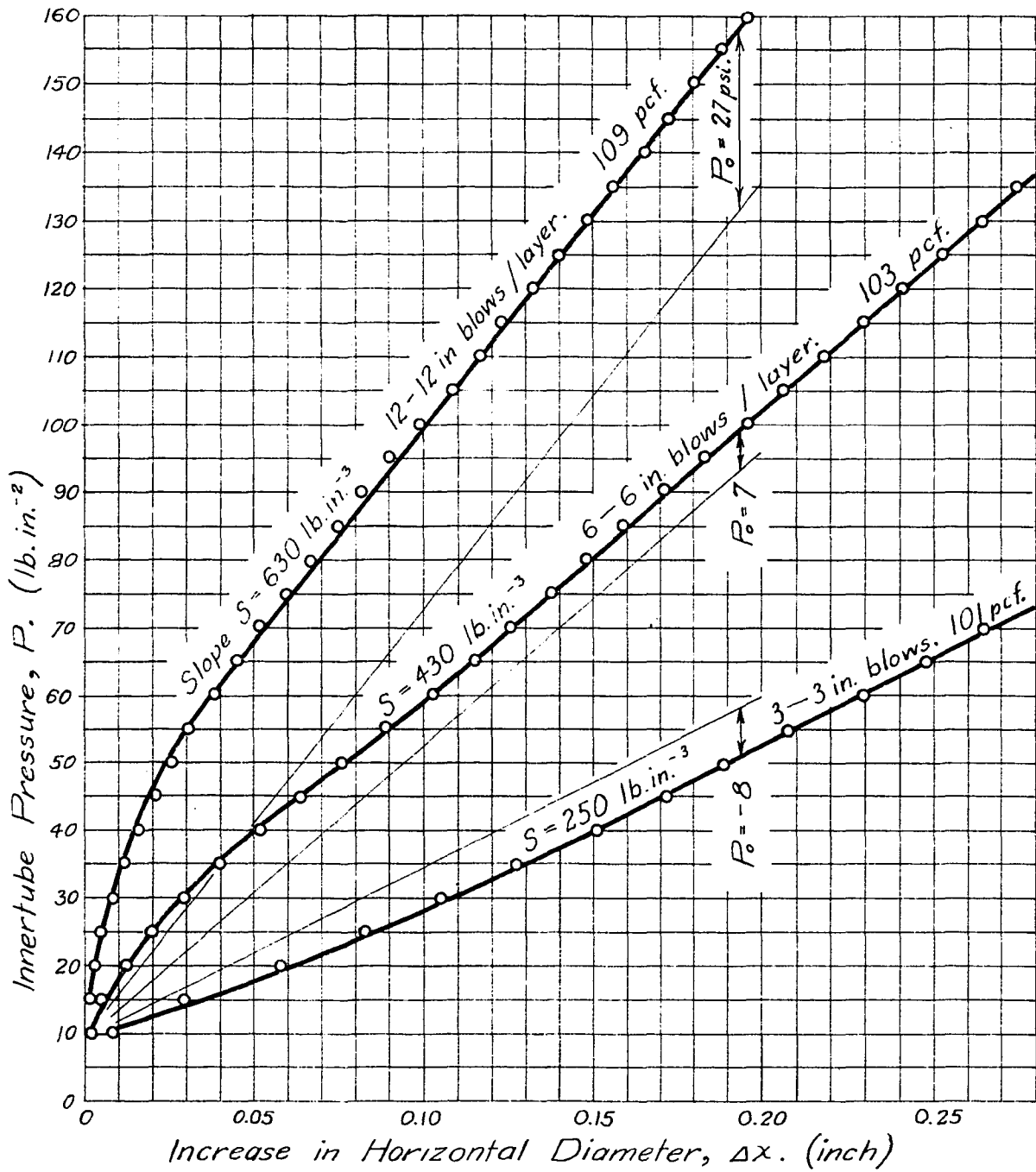


Figure 24. Load-deflection diagrams for clay at varying degrees of compaction using small model cell and 3 7/8 in. diameter pipe with 0.011 in. wall thickness (see Appendix F for soil identification)

voids divided by the volume of the solids and is a measure of soil density. It is well recognized that if soil (particularly sand) is placed in a shearing device in a very loose state (or high void ratio), the volume tends to decrease as shearing proceeds. On the other hand if the soil is placed in a dense state (or low void ratio), the volume tends to increase as shearing proceeds (3, p. 118). Now if the volume is held constant during the shearing process, the load required to cause shear rises markedly for soil in a dense state whereas the load is slow to rise if the soil is loose. It follows that there exists a definite void ratio at which there is neither increase nor decrease in volume during shear. Such a void ratio is defined as critical void ratio. This phenomenon explains the curved lower portions of the load-deflection diagrams. In the pipe-fill system the soil within the boundaries of relative soil displacement may be considered confined so that volume increase is strongly resisted. Under these conditions if the soil is dense the shearing loads (load pressures, P) rise rapidly at first with respect to deformation. As the loads increase the resistance to shear is overcome and at the same time some volume adjustment occurs so that a critical void ratio is reached and the load-deflection diagram continues as a straight line. See the load-deflection diagrams for six 6-inch blows per layer and twelve 12-inch blows per layer for sand in Figure 22. The

plot for three 3-inch blows per layer is more nearly straight throughout its entire length. Apparently the void ratio at three 3-inch blows per layer is close to a state of critical compaction. There is no doubt but what the plot for uncompacted sand is below this critical compaction since the straight line portion intersects the P axis at a negative value. It is interesting that the initial portion of the curve extends much further to the right than do any of the other curves for sand. This is reasonable since the soil within the zone of relative soil displacement decreases in void ratio (volume). Adjustment of the confining soil is not forced as in the case of dense sand, which increases in volume, so the resistance to shear increases very slowly until the vertical compression generally reduces the density of the confining soil to the same value as the soil which is being sheared. Thereafter the load-deflection plot continues as a straight line.

The term critical compaction is used instead of critical void ratio in the above explanation because there are many different conditions under which critical void ratio may be defined (21, p. 354-359). In the case of the pipe-fill system the continually varying relationship of loads and void ratios makes it impossible at present to say which of the existing definitions might apply. It would appear that additional work is necessary either to relate the critical

compaction for the pipe-fill system to an existing definition of void ratio, or to develop a new definition and new methods for evaluating the critical compaction. In this report confusion can be prevented if the term critical compaction is used to define the state of compaction at which the load-deflection plot has no initial curved portion and no P-intercept. The void ratios at no volume change (straight line portion) in other load-deflection diagrams can then be defined as critical void ratios. They are based on different conditions of soil pressure as evidenced by the fact that the slopes are all different.

Thus far it has been shown how the degree of compaction influences the load-deflection diagrams both by varying the slope and the P-intercept. Equation 18 shows that of these two the slope is the major factor affecting e_r . The P-intercept can affect e_r only indirectly and to the extent to which it alters C_1 in Equation 18.

It is not within the scope of this project to describe the extent to which compaction influences e_r . More than likely subsequent investigations will show that void ratio rather than degree of compaction is a better criterion. A rather extensive program for testing would be necessary to adequately describe the influence of compaction. Shorter methods do not appear forthcoming. No new principles nor equipment would be needed in such a program, however.

C. Influence of Pipe Wall Stiffness on the Modulus of Soil Reaction

The influence of pipe wall stiffness, EI , on the modulus of soil reaction may be investigated by using Equation 18 as a starting point. Resolving for e_r , Equation 18 becomes

$$e_r = 1.36 C_1 D T S - 131.2 \frac{EI}{D^3} . \quad \text{Eq. 19}$$

A series of load-deflection tests were run under identical conditions except for EI which was varied as shown in Figures 25 and 26. For tests on Saint Peter sand $D = 3.875$ inch, $T = 0.18$. See "Effect of the Cell Walls on the Modulus of Soil Reaction ". Values for S , the average slope of the load-deflection diagram, vary with EI as shown in Figure 25 and Table 2. Since C_1 can theoretically vary from about 0.7 to 1.0 two solutions of e_r in terms of EI are shown in Table 2. Column 6 is based on the assumption that $C_1 = 1$ and Column 7 is based on the assumption that $C_1 = 0.7$. Both Columns 6 and 7 show that for sand the effect of pipe wall stiffness on e_r is negligible within the accuracy of measurements involved. Some question still remains as to which value of C_1 to use. Indeed a value of $C_1 = 0.65$ appears to give the most consistent value of e_r . Within the accuracy of measured quantities all values of C_1 considered in Table 2 substantiate the independence of e_r from EI , but they all give different values of e_r . There is some justification for

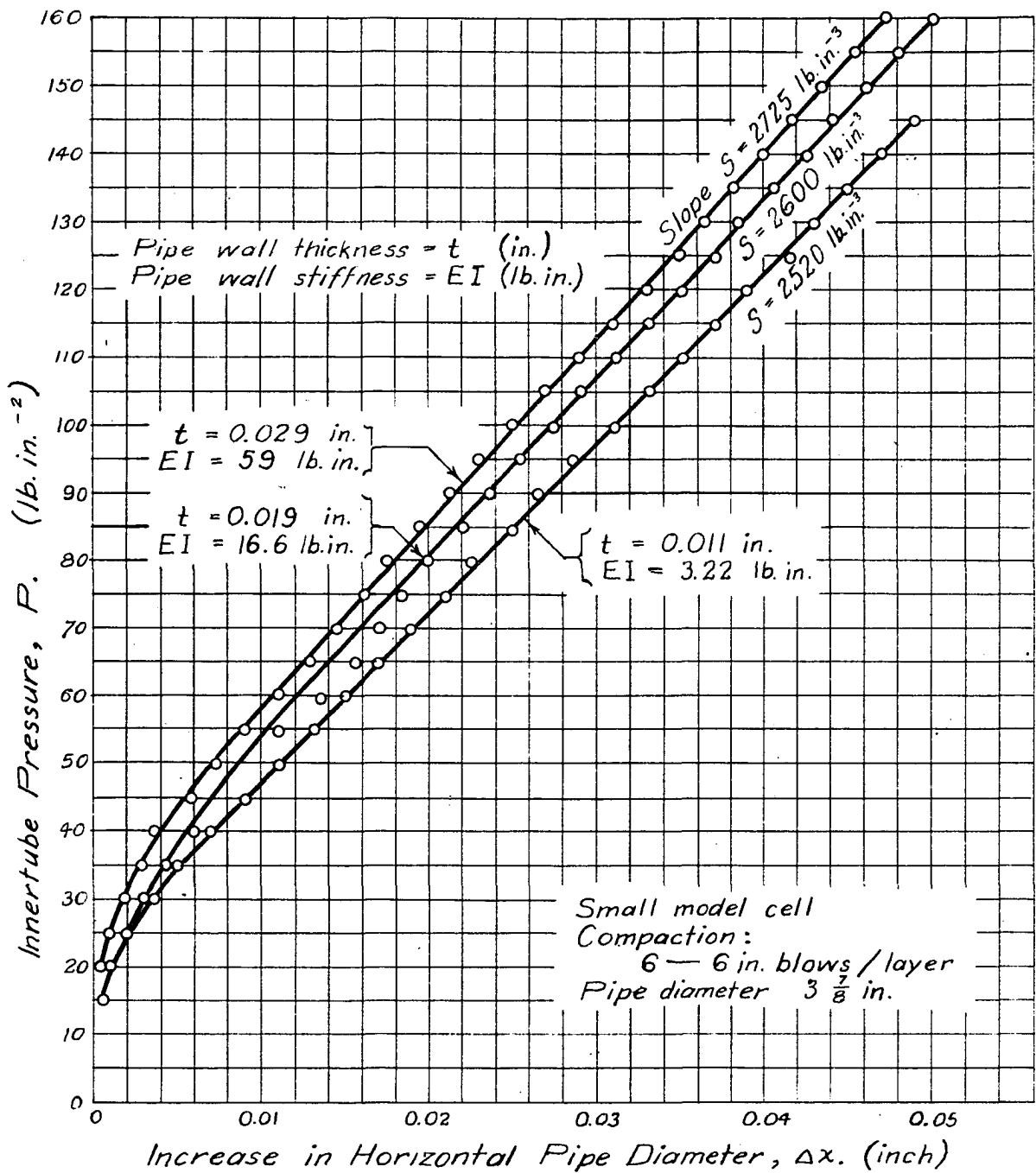


Figure 25. Load-deflection diagrams for white silica sand (Saint Peter Formation) at varying values of EI

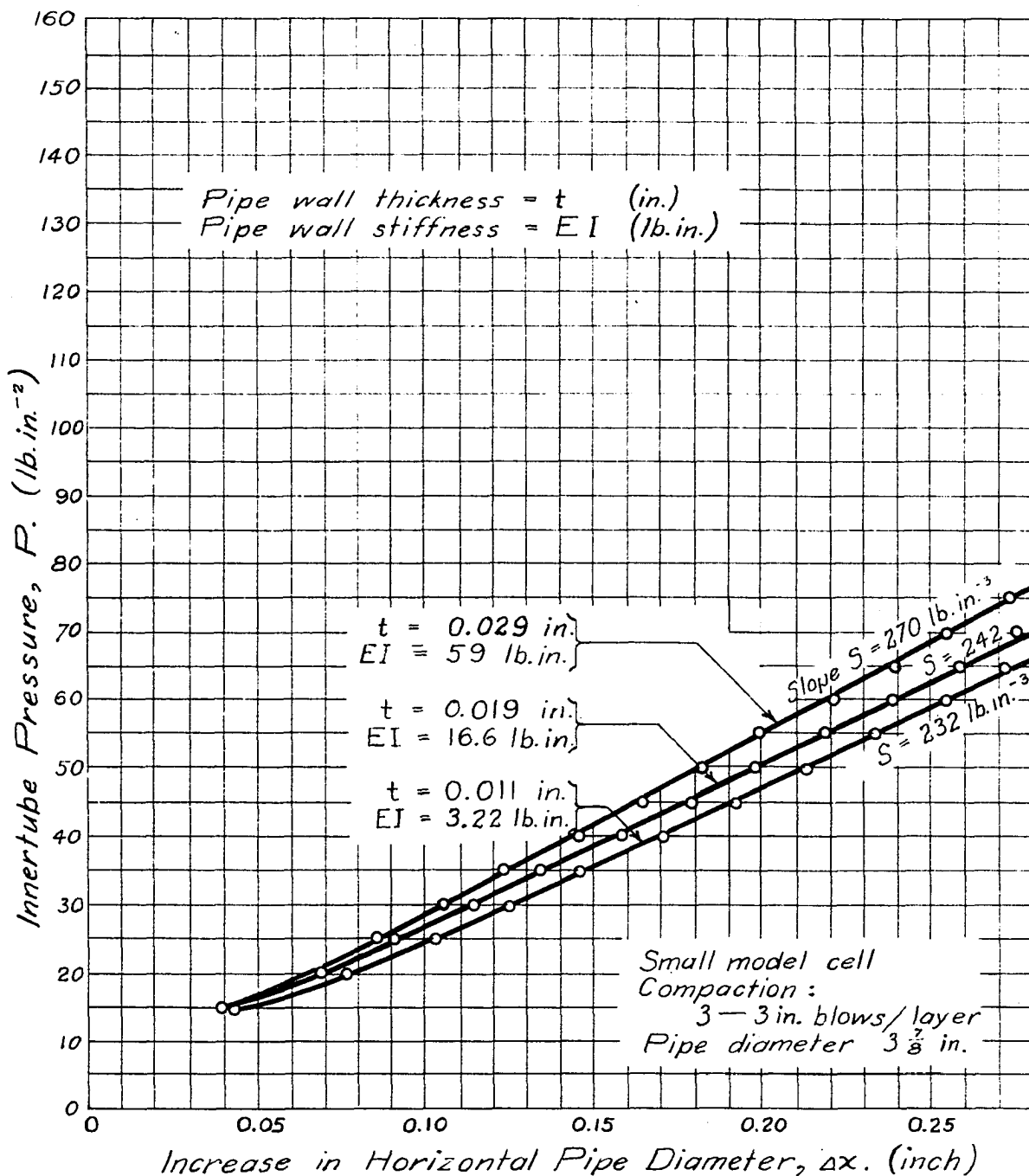


Figure 26. Load-deflection diagrams for loess at varying values of EI

Table 2. Determination of e_r as a function of pipe stiffness for sand compacted by six 6-inch blows per layer

1 ^a t (inch)	2 ^b EI (lb.in.)	3 ^c S	4 ^{c,d,e,f} 1.36 $C_1 D^3 S$ ($C_1 = 1$) (lb.in. ⁻²)	5 ^{b,e} 130 $\frac{EI}{D^3}$ (lb.in. ⁻²)	6 ^{d,g} e_r ($C_1 = 1$) (lb.in. ⁻²)	7 ^{d,g} e_r ($C_1 = 0.7$) (lb.in. ⁻²)	8 ^{d,g} e_r ($C_1 = 0.65$) (lb.in. ⁻²)
0.011	3.22	2520	2390	10	2380	1780	1550
0.019	16.6	2600	2460	40	2420	1810	1560
0.029	59	2725	2580	130	2450	1810	1550

^at = pipe wall thickness

^bEI = pipe wall stiffness factor per unit length

^cS = slope of load-deflection diagram

^d C_1 = constant

^eD = pipe diameter = 3 7/8 inches

^fT = 0.18 (See "Model Boundary Effects")

^g e_r = modulus of soil reaction

accepting the assumption that C_1 be unity. In the first place it can be forgotten. In the second place since sand placed with any compaction whatsoever is relatively incompressible as compared with other soils, the amount of settlement at any level below the plane of equal settlement should be about the same at all points including points above the pipe. If this is true the settlement ratio approaches zero and $C_1 = 1$. As a matter of fact most design of flexible pipe at the present is based on the simplifying assumption that $C_1 = 1$. For example, Kelley (7, p. 365) assumes that the load on a flexible pipe culvert is

$$W_c = \gamma DH.$$

When compared with Marston's load theory that

$$W_c = C \gamma D^2, \quad \text{Eq. 2}$$

it is evident that C must be equal to $\frac{H}{D}$. But according to Figure 14, C can only equal $\frac{H}{D}$ if $C_1 = 1$. It would be convenient if C_1 could be set at unity for all soils and be defined according to Equation 19 with $C_1 = 1$. Unfortunately an investigation of loess showed that C_1 cannot be set at unity.

Figure 26 shows three load-deflection diagrams for loess compacted at three 3-inch blows per layer. This is close to critical compaction for loess. Again Equation 19 is used to evaluate e_r with results as indicated in Table 3. In this case, however, the assumption that $C_1 = \text{unity}$ does not give consistent values for e_r . See Column 6. Rather, consistent

Table 3. Determination of e_r as a function of pipe stiffness for loess compacted by three 3-inch blows per layer

1 ^a t (inch)	2 ^b EI (lb.in.)	3 ^c S	4 ^{c,d,e,f} 1.36 C ₁ DTS (C ₁ = 1) (lb.in. ⁻²)	5 ^{b,e} 130 $\frac{EI}{D^3}$ (lb.in. ⁻²)	6 ^{d,g} e _r (C ₁ = 1) (lb.in. ⁻²)	7 ^{d,g} e _r (C ₁ = 3.2) (lb.in. ⁻²)
0.011	3.22	232	244	7	235	776
0.019	16.6	242	255	37	218	779
0.029	59	270	286	132	154	779

^at = pipe wall thickness

^bEI = pipe wall stiffness factor per unit length

^cS = slope of load-deflection diagram

^dC₁ = constant

^eD = pipe diameter = 3 7/8 inches

^fT = 0.20 (See "Model Boundary Effects")

^ge_r = modulus of soil reaction

values for e_r are produced only when $C_1 = 3.2$ as indicated in Column 7. If C_1 can be assigned such a value, then e_r is independent of EI just as for sand. There is little doubt that two or more load-deflection tests are necessary if values of C_1 are to be determined for any given soil at a given degree of compaction.

The results of the present investigation of C_1 show method only. They are not numerically accurate. For example, the slopes of the load-deflection diagrams are based on only one test for each. Figures 27, 28, and 29 show that for sand there is a spread of values if more than one test is carried out under supposedly identical circumstances. The duplicate tests represented in Figure 28 show excellent agreement, but the triplicate tests of Figures 27 and 29 show some discrepancy. The second and third tests in both figures were performed at later dates at which time moisture contents and testing techniques might have varied slightly as indicated by the weight of sand in each case. The first test in each figure was replotted from the sequence of tests in Figure 25. Actually the discrepancies between the second and third tests in Figures 27 and 29 represent more nearly the average discrepancies in load-deflection diagrams for duplicate tests on sand. Duplicate tests on loess generally showed even less discrepancy. One who has tested the mechanical properties of soil will recognize that such replicability of tests results

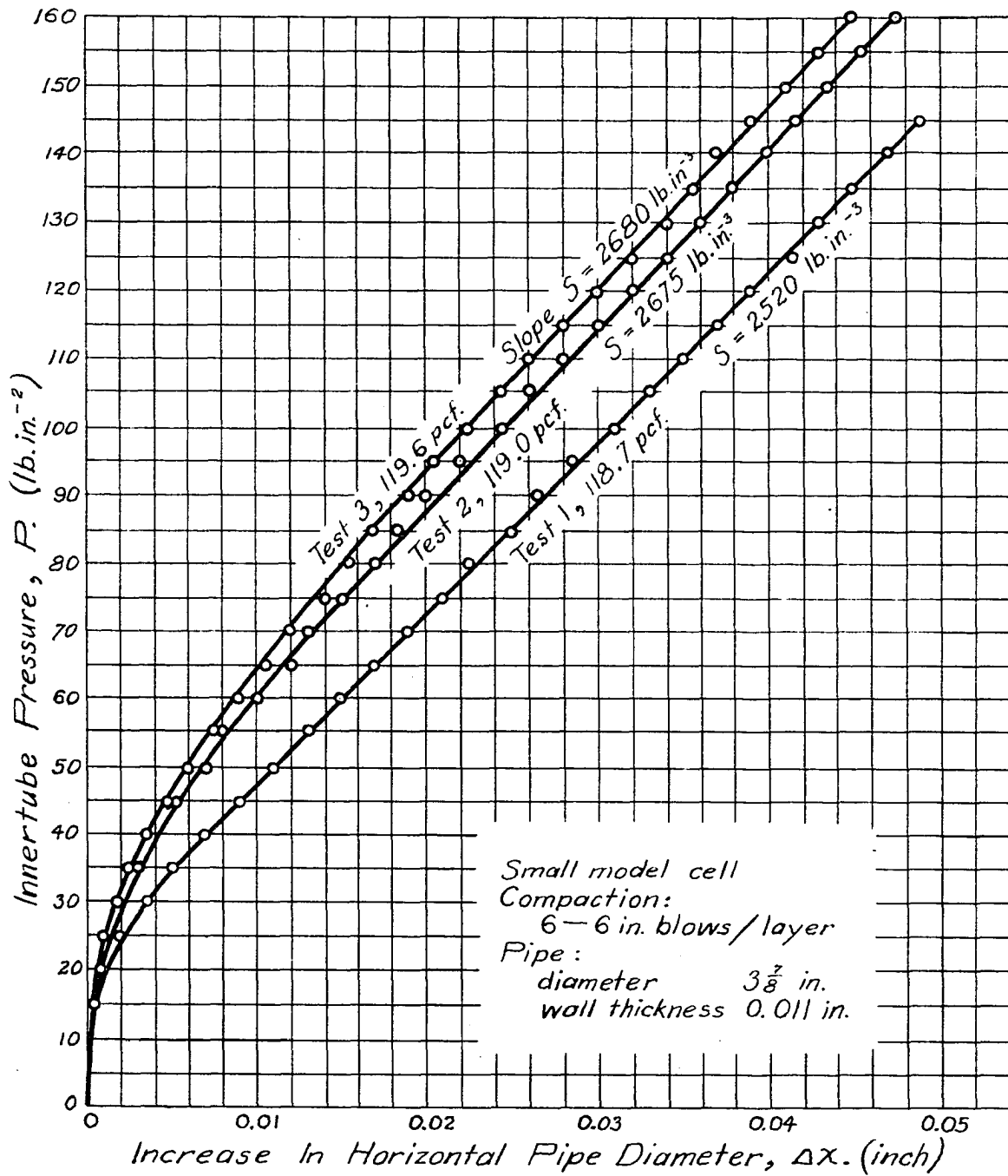


Figure 27. Load-deflection diagrams for triplicate tests on white silica sand (Saint Peter Formation) with pipe wall thickness of 0.011 in.

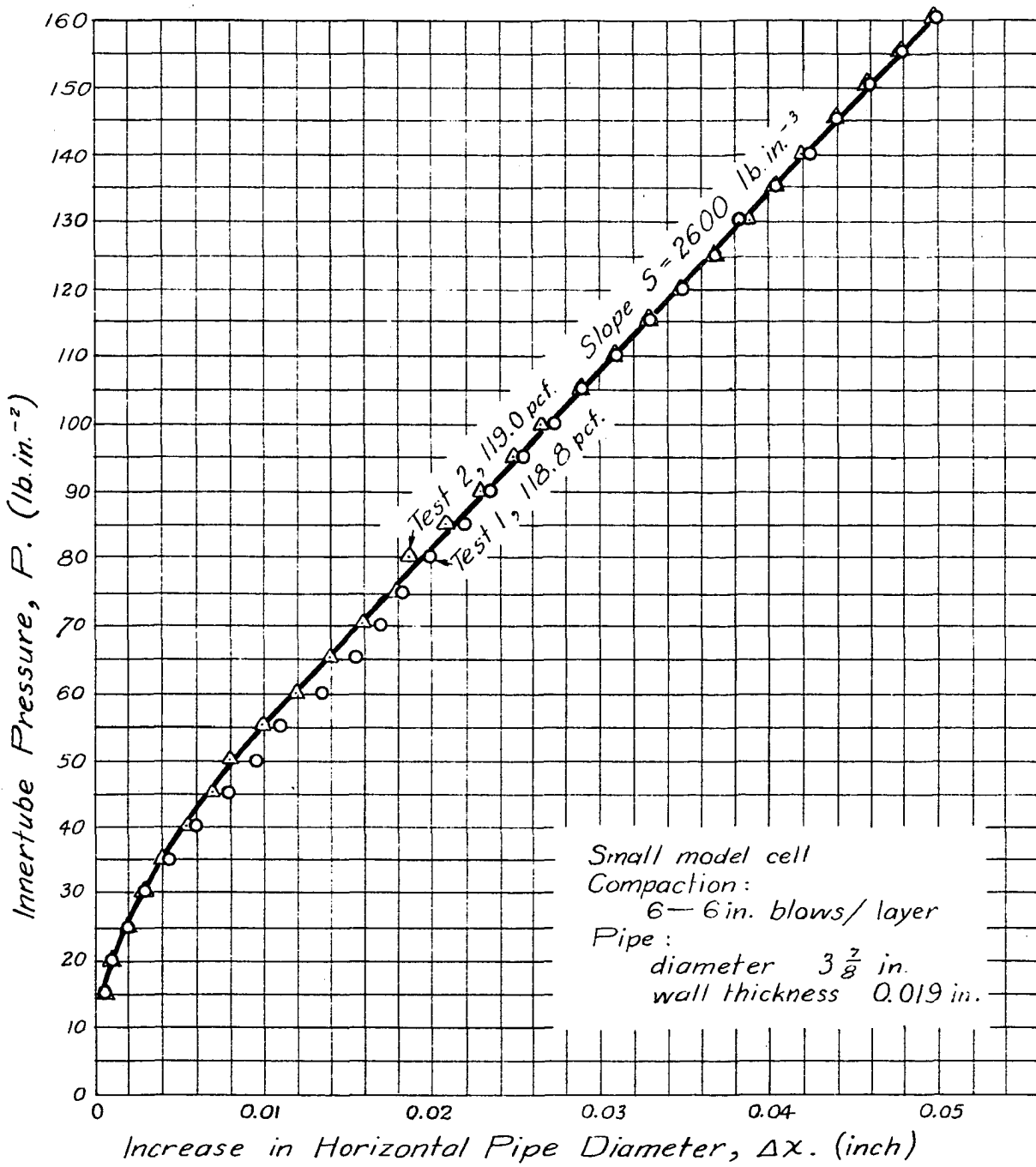


Figure 28. Load-deflection diagrams for duplicate tests on white silica sand (Saint Peter Formation) with pipe wall thickness of 0.019 in.

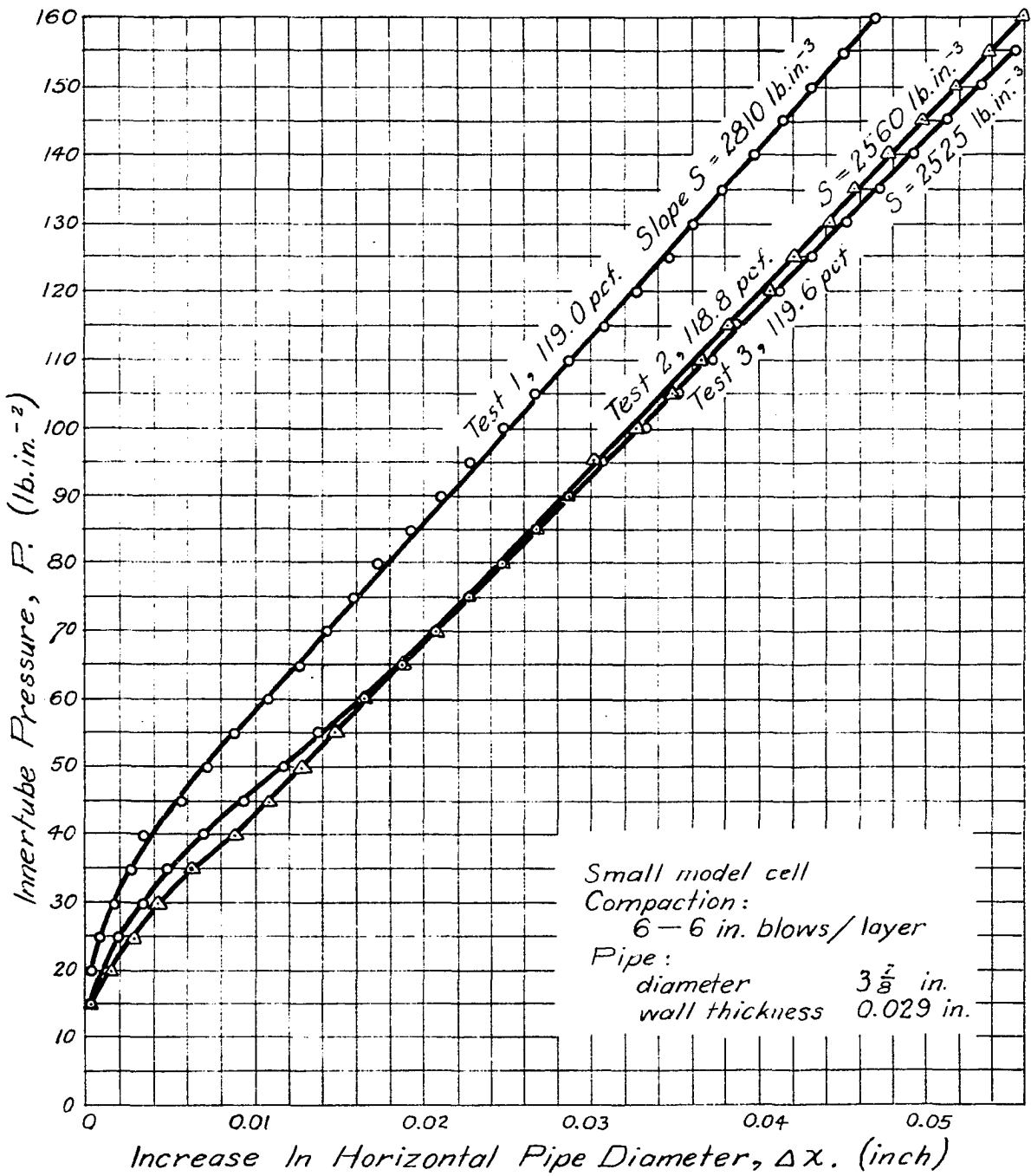


Figure 29. Load-deflection diagrams for triplicate tests on white silica sand (Saint Peter Formation) with pipe wall thickness of 0.029 in.

in soils is remarkably good. Scattering of points on any individual load-deflection diagram is much less than would be expected and suggests that even better replicability could be realized if the techniques of compaction and pipe placement were refined. The basic purpose of any such refinement would be to evaluate C_1 . Table 2 shows, for example, that C_1 is 0.65 for sand based on the sequence of tests of Figure 25, but using some other combination of slopes from Figures 27, 28, and 29, the value for C_1 would be entirely different. In the case of sand, however, further refinement of techniques could scarcely be justified since the assumption that $C_1 = 1$ leads to sufficiently consistent values for e_r . In the case of loess and clay, refinement of techniques may be justified since C_1 must be evaluated.

The important conclusion from this discussion is that e_r is essentially independent of EI provided that an appropriate value for C_1 is established. As emphasized before, this statement must preclude the incomplete projection condition under which the pipe may be so stiff as to affect e_r . C_1 appears to be a function of soil characteristics only, and may be evaluated by series of carefully controlled load-deflection tests.

D. Model Boundary Effects

1. Effect of friction of the cell walls on the vertical soil pressure

The load pressure in the innertubes was not the same as the vertical soil pressure at the level of the top of the pipe. As a matter of fact most of the load pressure was lost through friction against the walls of the model cells. Values of the vertical reaction pressure at various points in the soil of the small cell as a function of load pressure, P , are shown in Figures 30, 31, and 32. In every case the relationship between reaction pressure and load pressure is linear. The slope of the line is called the transmission ratio and is designated by T . This linearity immediately eliminates any possibility that friction in the model cells caused the curved portions of the load-deflection diagrams. From the tests on loess, variation of transmission ratio, T , with respect to height of the soil cover, z , was plotted as shown on Figure 33. It is evident from this diagram that the transmission ratio at the top of a $3 \frac{7}{8}$ inch diameter pipe at the center of the small model cell is $T = 0.20$. For sand the transmission ratio is slightly less, due, no doubt, to a higher friction angle of the sand against the walls of the model cell. By a simple proportion, a value of T for

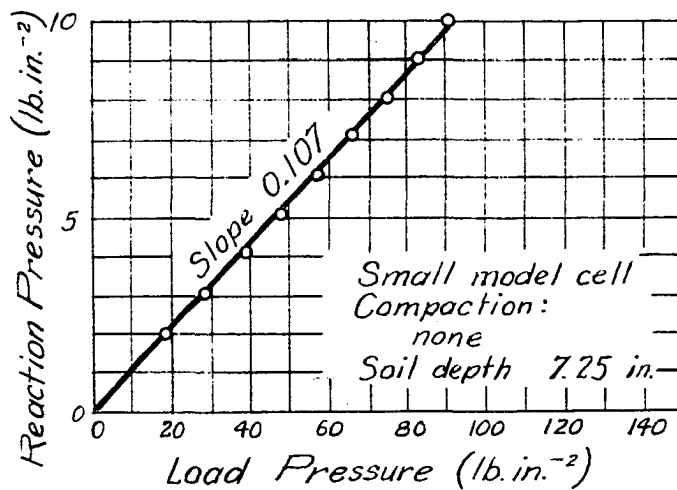
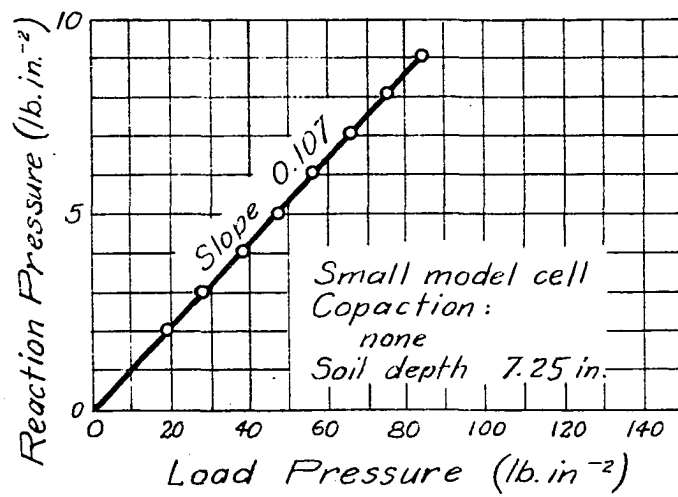


Figure 30. Plots of duplicate tests for reaction pressure as a function of load pressure for white silica sand (Saint Peter Formation) in small model cell

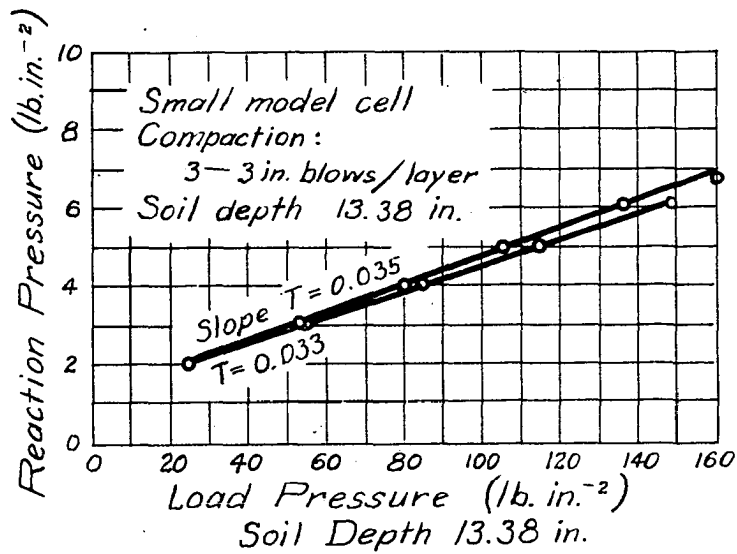
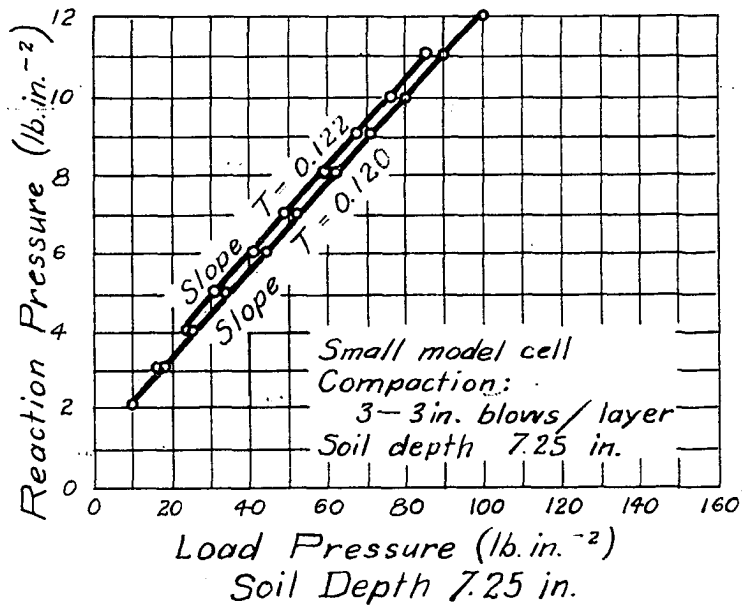


Figure 31. Plots of duplicate tests for reaction pressure as a function of load pressure for loess at varying soil depths

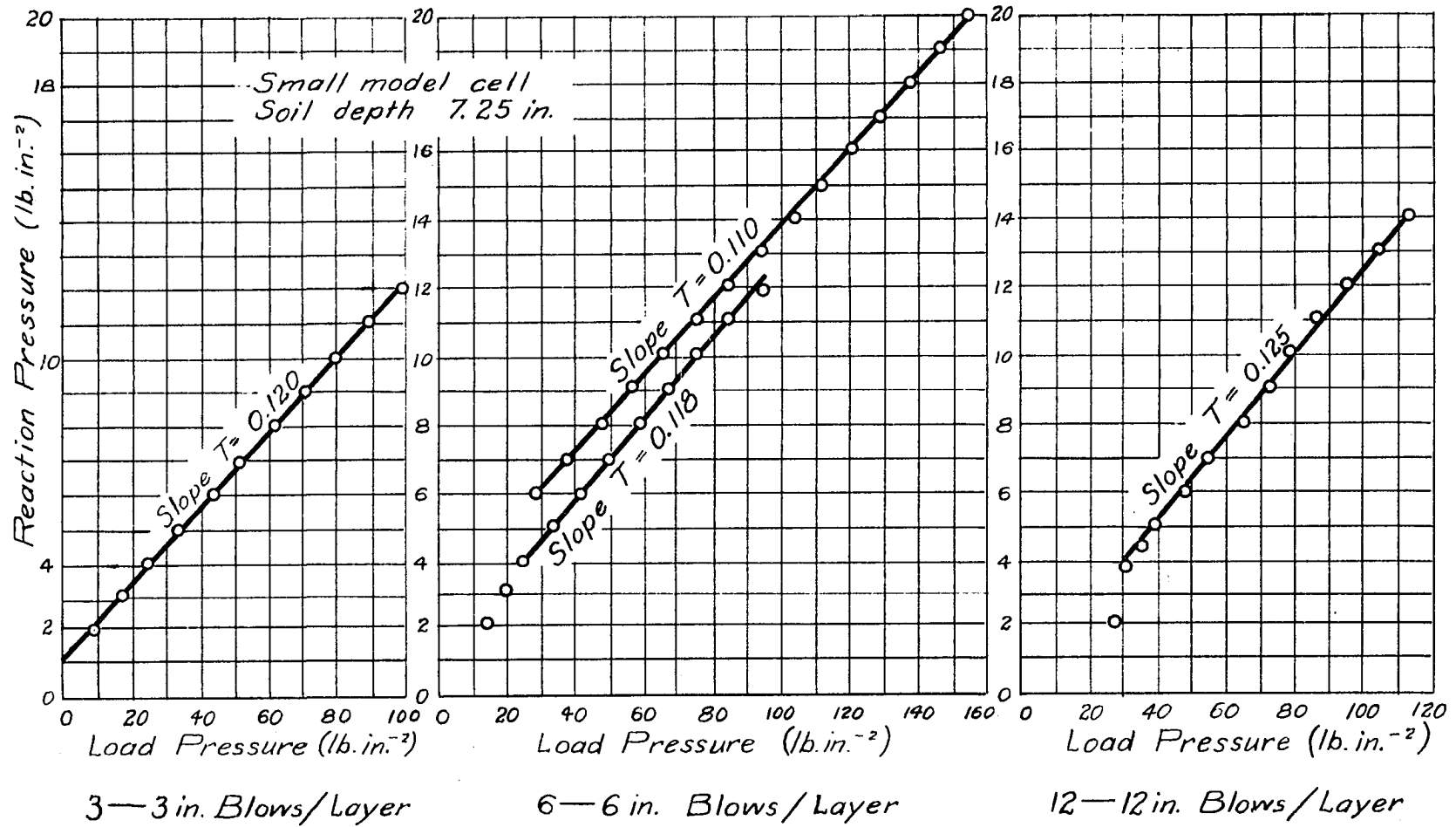


Figure 32. Plots of reaction pressure as a function of load pressure for loess at varying degrees of compaction

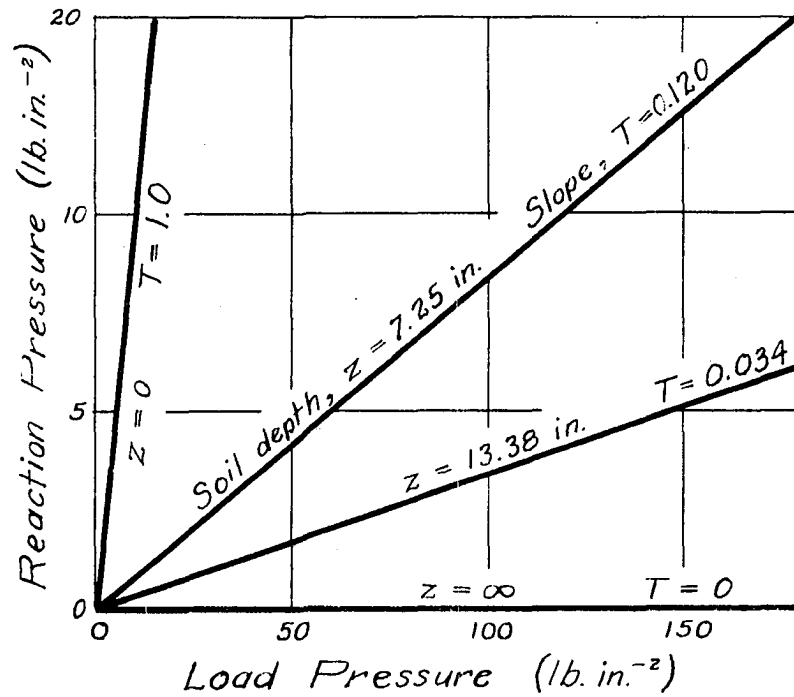


Figure 33a. Reaction pressure as a function of load pressure for loess at varying soil depth

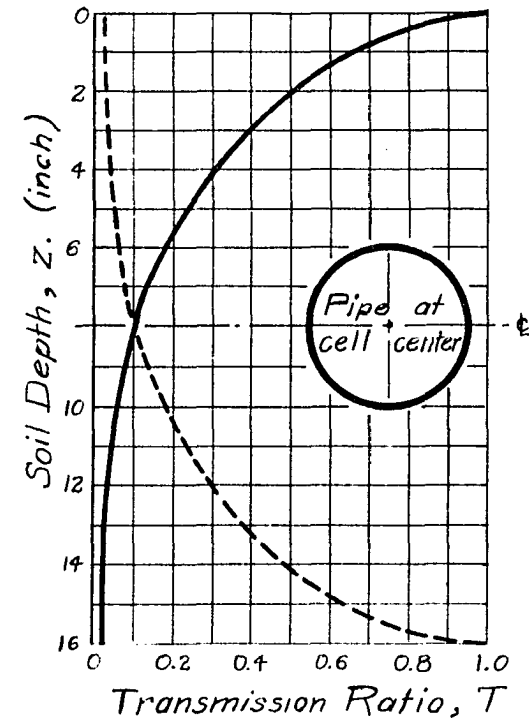


Figure 33b. Transmission ratio as a function of soil depth for loess in the small model cell showing also an inverted curve (dotted) for load pressure at bottom of cell

sand at the same location would be $T = 0.18$.

2. Effect of the cell walls on the modulus of soil reaction

A series of X-ray tests were conducted on loess in order that boundaries of relative soil displacement might be located. Figures 34 and 35 were made up from two sets of X-ray photographs on loess compacted by three 3-inch blows per one inch layer. Each X-ray photograph of each set was made at a different load pressure (innertube pressure). The circled points represent the average positions of lead shot at the load pressure indicated. The location of the shot on each X-ray photograph of a set were superimposed on tracing cloth. To average the shot positions the tracing cloth was folded on the vertical pipe axis and mean positions of the shot were plotted, then the tracing cloth was folded on the horizontal pipe axis and the mean of the mean positions were plotted. Figures 34 and 35 are tracings of the results.

Figure 36 is a similar plot for a control case in which no pipe section was embedded. By subtracting the displacements of Figure 36 from the displacements of Figure 35 the relative soil displacement could be plotted. Such a plot is shown in Figure 37 where the rectilinear grid system represents the initial position of the soil before any load pressure is applied. As load pressures of 80 psi, and 160 psi. are consecutively applied the relative soil displacement is

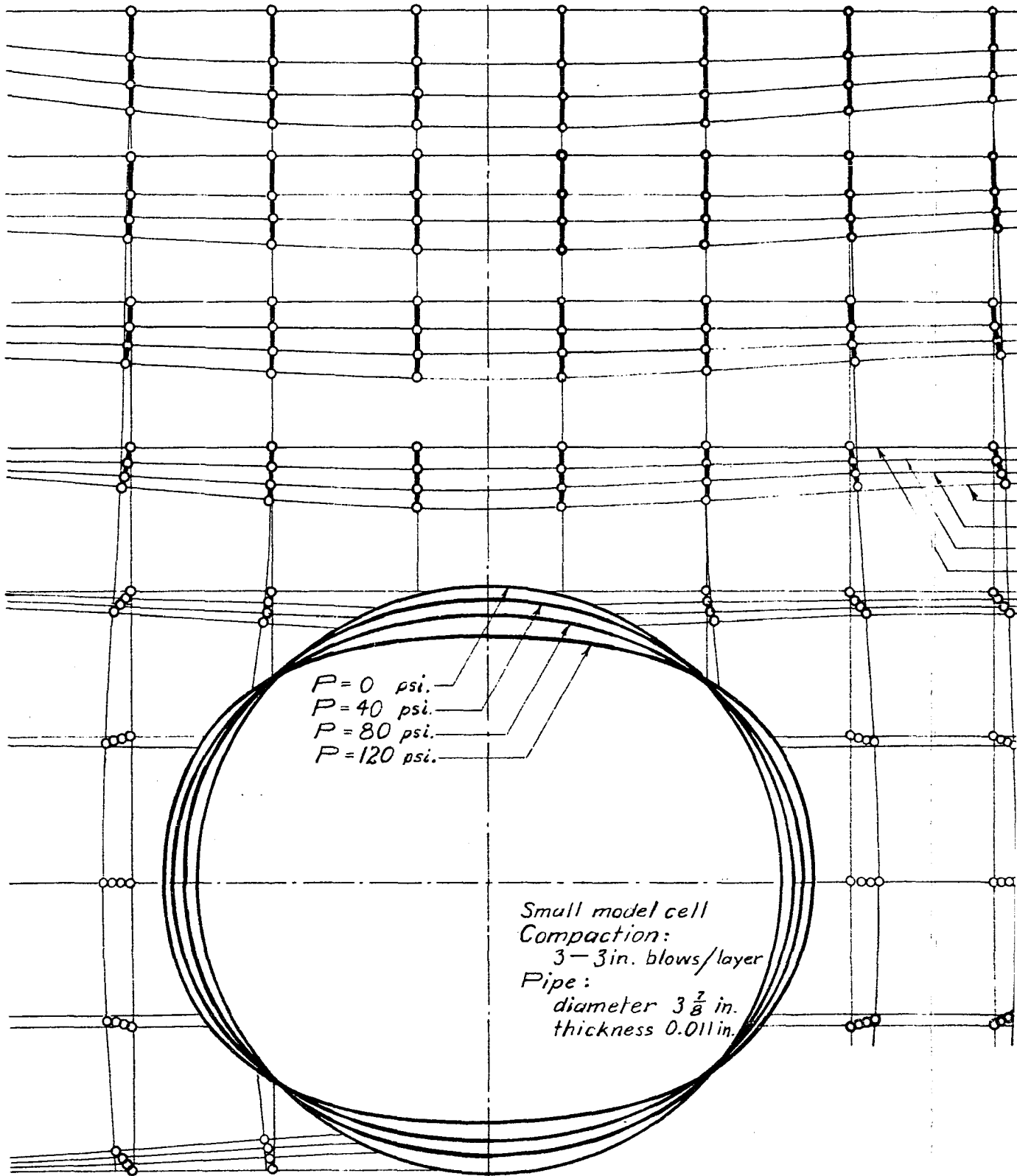


Figure 34. Positions of lead shot in loess at various load pressures if pipe wall thickness is 0.011 in.

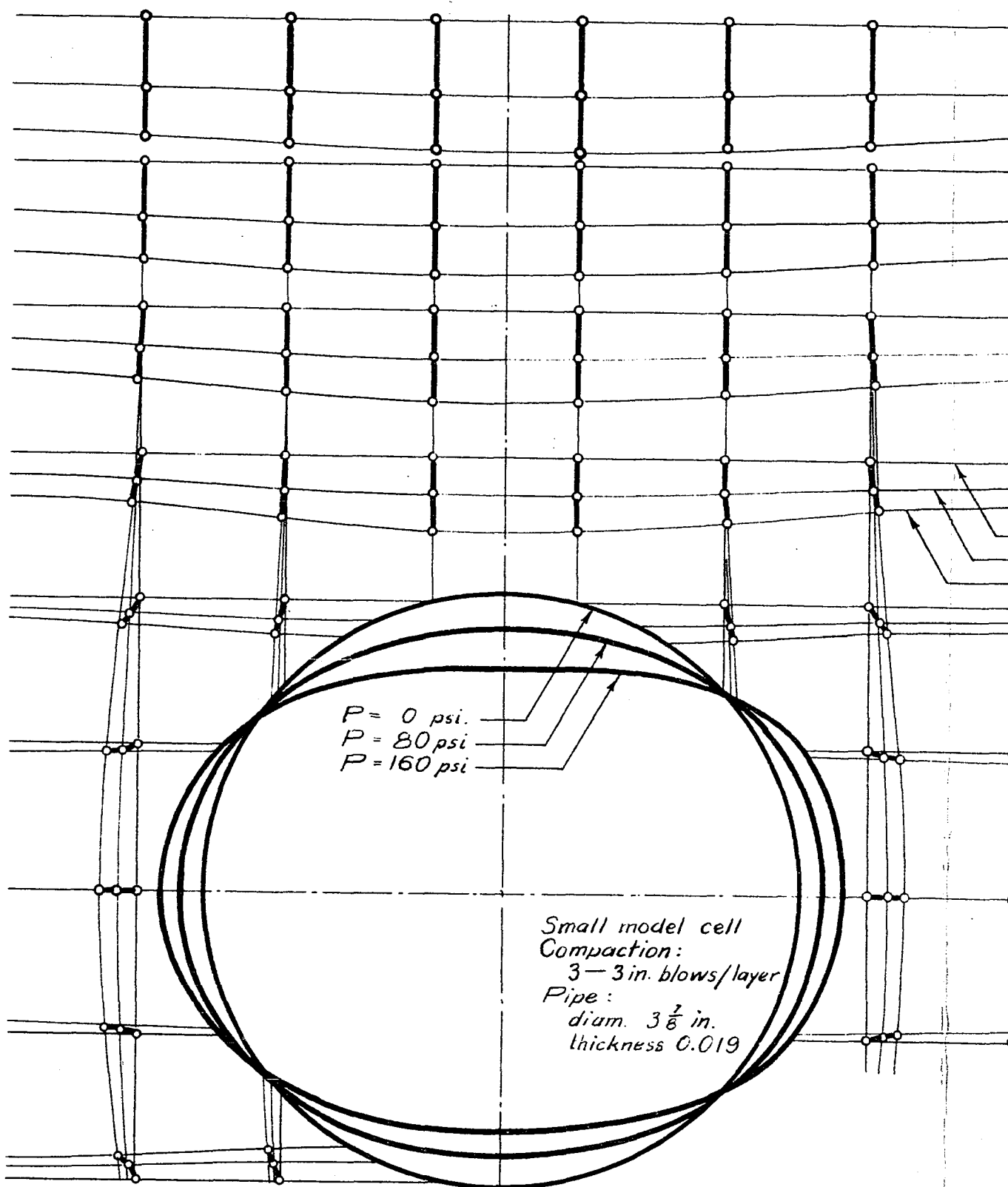


Figure 35. Positions of lead shot in loess at various load pressures if pipe wall thickness is 0.011 in.

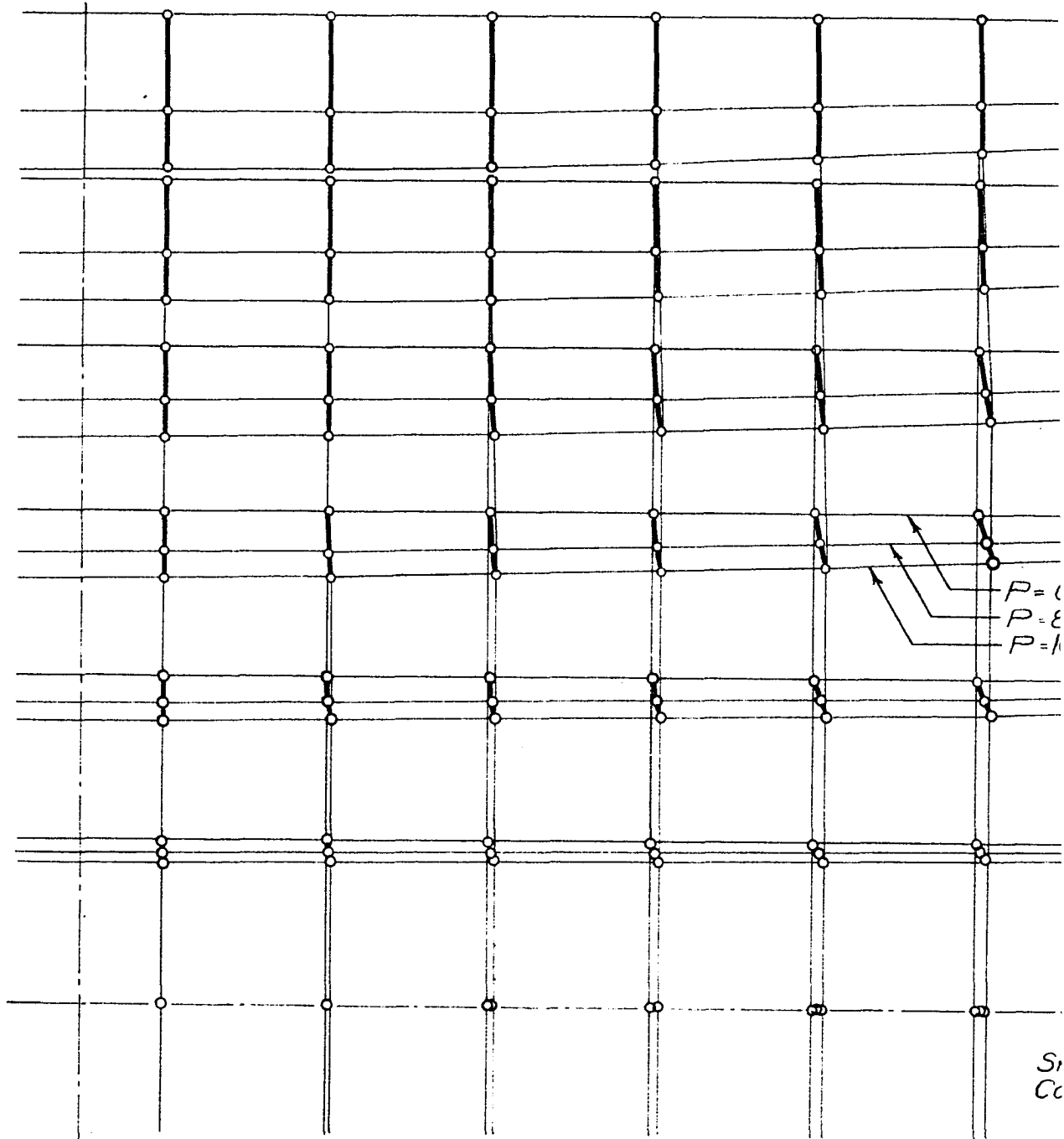


Figure 36. Superimposed positions of lead shot from X-ray photographs for loess with no model pipe (control case) at varying load pressures, P

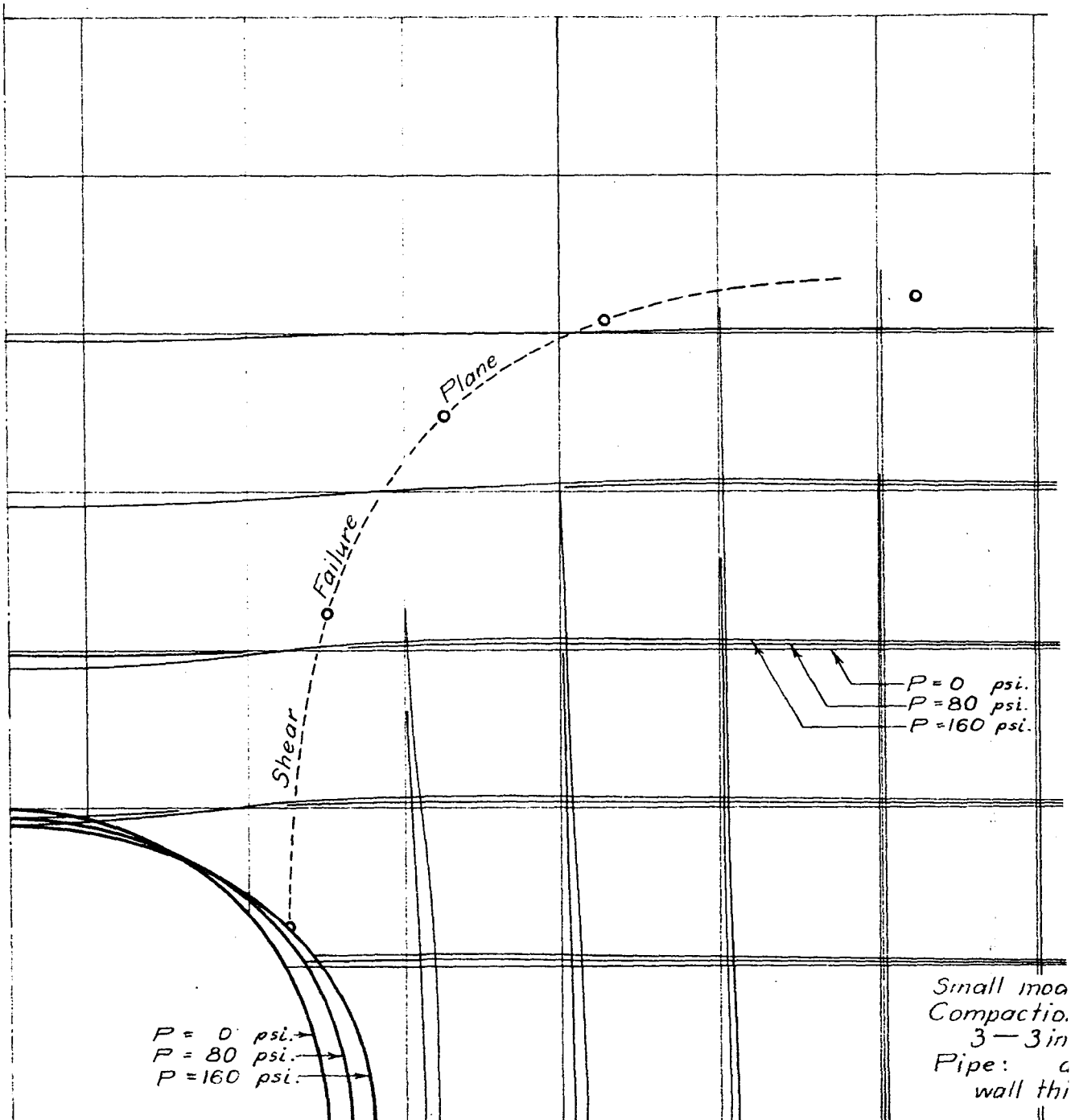
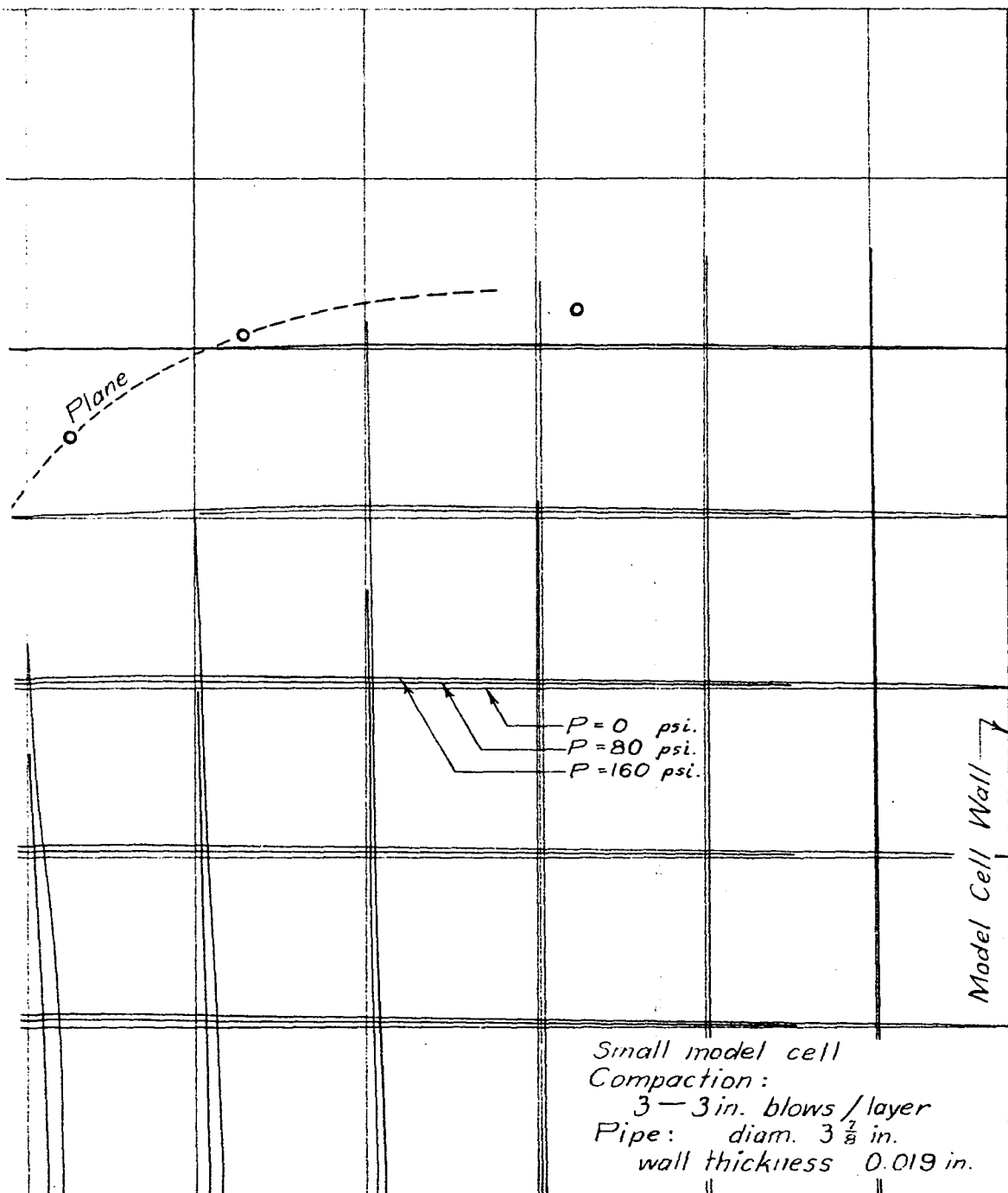


Figure 37. Relative soil displacement of loess at varying load pressures, P , showing also the shear failure plane (dotted)



placement of loess at varying
 P , showing also the shear
 plotted)

as shown by the two curvilinear grid systems. Where only one curvilinear line is shown, it is for the 160 psi, pressure only.

From Figure 37 it may be reasoned that the Y-boundaries of relative vertical soil displacement (planes of equal settlement) are probably within the model cell walls for low load pressures, but that they increase and exceed the model cell boundaries at high pressures. From this information alone it would be difficult to arrive at any exact location of the plane of equal settlement for any given pressure.

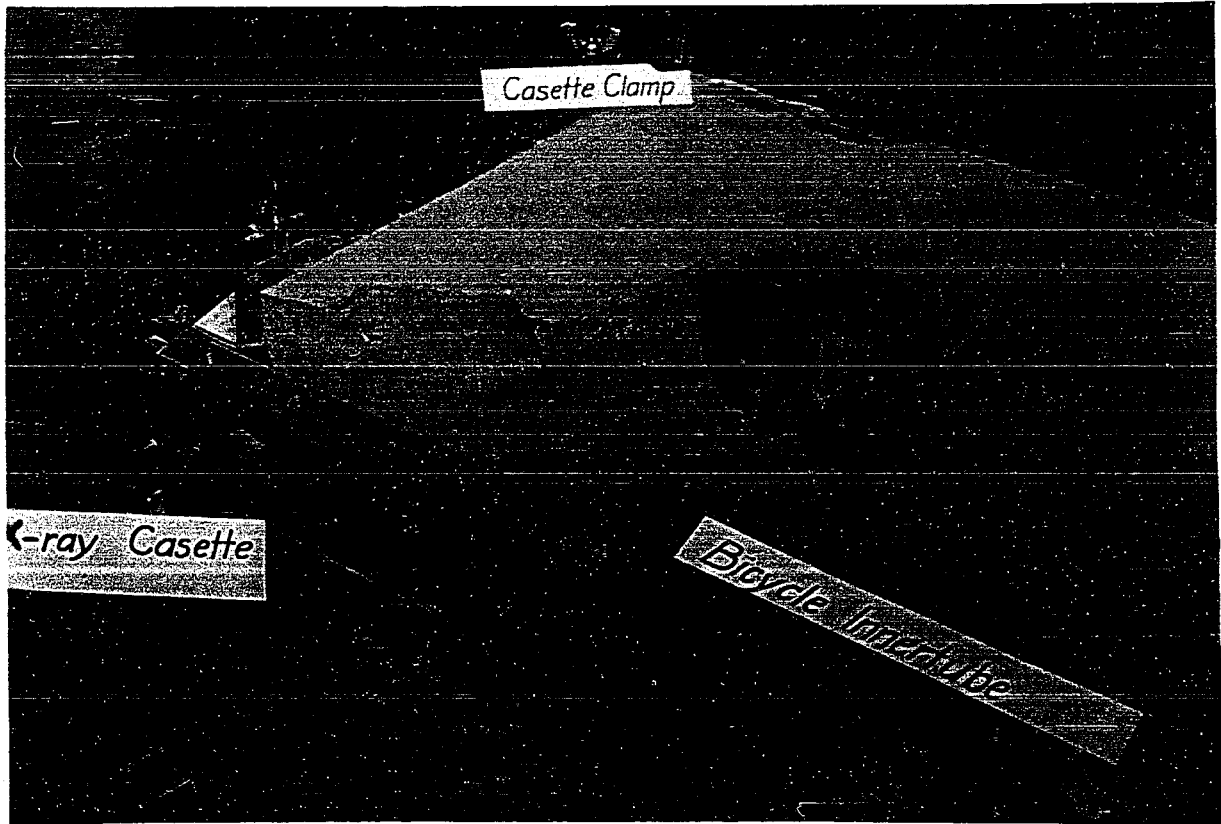
X-boundaries of relative horizontal soil displacement are probably at or outside of the model cell walls. The magnitude of relative displacement from the side of the pipe to the wall of the cell is inversely proportional to the distance from the pipe. This essentially is true for pressures of both 80 psi. and 160 psi. Evidently there is not so much displacement of the X-boundaries of relative soil displacement with respect to pressure as there is displacement of the plane of equal settlement.

Of special interest in Figure 37 is the boundary of the generalized shear region. It is shown dotted. After each test on loess the soil to the left of the dotted line was comparatively soft while the soil to the right was very hard. If the cell was tipped so that the soil fell out, it would

tend to fracture along the dotted line. The dotted line was actually plotted from an average of two such fractures in loess. Figure 38 is a photograph of such a failure plane. The photograph does not do credit to the well defined position of the shear plane since some slipping occurred as the aluminum plate was removed for the picture. Referring again to the dotted shear boundary on Figure 37, the loose soil to the left was probably not any denser than for initial compaction conditions of three 3-inch blows per layer. The density could even have been less since the region was in a state of general shear and since compaction of three 3-inch blows per layer was slightly above critical compaction. It is reasonable to believe that if the dotted line could be traced on out it would merge with the plane of equal settlement. Of course it would be difficult to trace on out since relative soil displacements become smaller the further they are from the pipe.

The X-ray photographs did not provide numerical results so two additional approaches were used to evaluate the effect of cell boundaries on ϵ_r . Tests were conducted on the large model cell using the $3 \frac{7}{8}$ inch pipe section of the small cell. See Figure 39. In this case there was little doubt that the plane of equal settlement was within the cell, and even the largest possible X-boundary spacing, according to Peck's bearing theory, was met. See Appendix E. The slope

Figure 38. Typical shear failure plane in loess



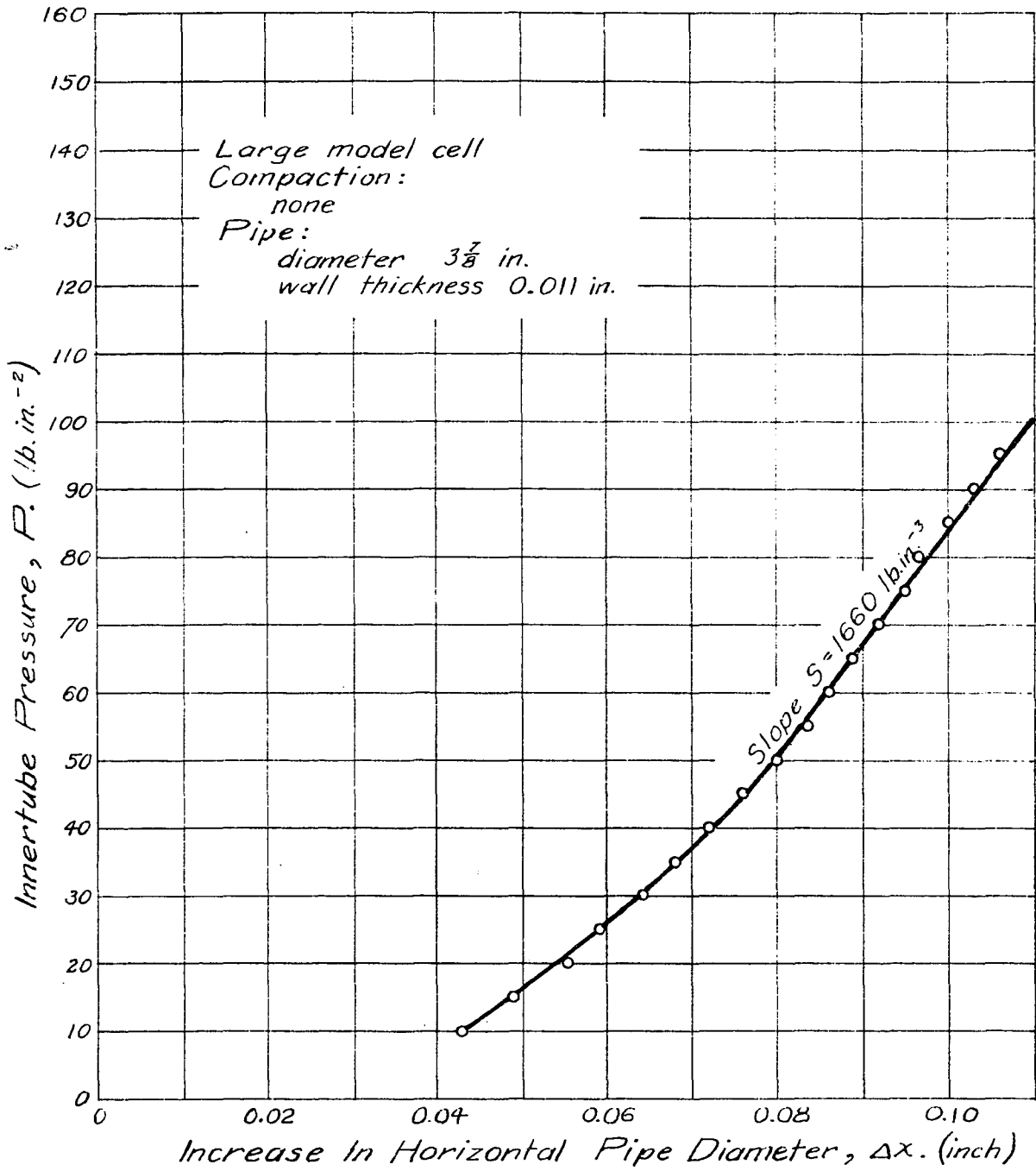


Figure 39. Load-deflection diagram for white silica sand (Saint Peter Formation) in large model cell using small pipe model

of the load-deflection diagram of Figure 39 is about 1600. With the same pipe section, tested under identical conditions in the smaller model cell, the slope is about 1200. See Figure 22. Applying Equation 19 and disregarding the negligible influence of the pipe wall stiffness, e_r would appear to be one-third larger in the large model. This difference can only be due to cell wall influence. If the large cell model includes boundaries of relative soil displacement, then for uncompacted Saint Peter sand in field installations, e_r is 1.33 times the value determined by the small model cell.

The second attempt to evaluate the effect of model cell boundaries on e_r was carried out by applying the vertical load pressure through rigid plates rather than innertubes. In this case the small model cell was used and loess was compacted at three 3-inch blows per layer. The use of rigid plates insured that the plane of equal settlement remained within the model cell. No allowance was made for the X-boundary effect. Figure 40 shows a load-deflection diagram for the test using rigid plates. e_r for the linear portion up to about 5.7 per cent deflection is 1055 lb. in.⁻² if C_1 is taken as 3.2. The other curve on Figure 40 is for a test under identical conditions but using the innertubes for applying the load. e_r for this curve is 771 lb. in.⁻². Interestingly enough the ratio of e_r 's for the two cases is 1.37. This is very close to the 1.33 evaluated by using the large model cell to determine boundary effects. It is

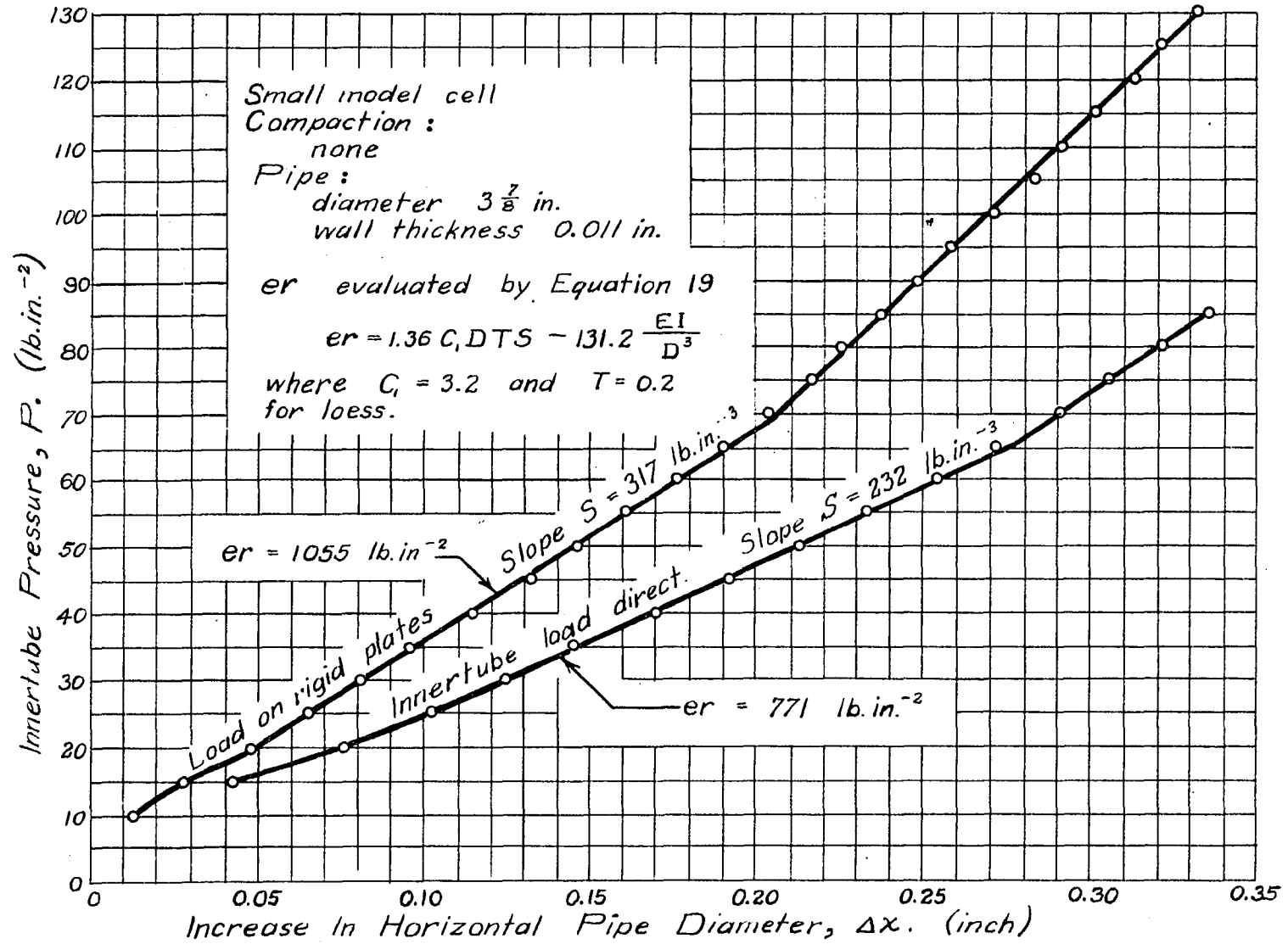


Figure 40. Comparison of slope-deflection diagrams for loess with the load on rigid plates and with the load pressure applied directly

possible, then, that the X-boundaries of the small model cell are adequately spaced at $L_x = 4.2D$ but that the Y-boundaries (or planes of equal settlement) should be spaced further apart. This presumption could only hold if the boundary effects are the same on both sand and loess.

There is some indication from the agreement between results that either the large model cell with the small pipe or the small model cell with the rigid plate loading might adequately simulate field conditions.

An inspection of Figure 40 reveals strong evidence that X-boundary effect may change at about 70 psi. load pressure. Above this point in both diagrams the curve takes on a different slope. This condition did not show up on the large model cell. However, on many tests on loess on the small model a rather definite break appeared in the load-deflection diagrams above 7 per cent deflection. More work should be done in this range of deflections.

Since the object of this project was the investigation of characteristics rather than the numerical evaluation of er, the slope of Figure 39 is not so important as the fact that it has the same general configuration as do the diagrams of Figure 22.

3. Effect of the model pipe on the modulus of soil reaction

Principles of similitude establish the fact that the shape of the pipe cross section should be the same for model

and prototype for a given pipe-fill system, but one source of concern is the possibility that the shape of the pipe might vary from an ellipse as assumed in the derivation of the Iowa Formula. Comparison of pipe shapes as a function of EI at $\Delta x = 10$ per cent of D is shown in Figure 41 which is taken from the X-ray photographs. The value of 10 per cent is used instead of 5 per cent because the deviations are exaggerated for visual comparisons. The Δx and Δy values have also been doubled to facilitate visual comparisons. Clearly the pipe does not remain elliptical during deformation nor does the vertical deflection equal the horizontal. The stiffer the pipe wall, the more nearly does the pipe remain elliptical and the more nearly equal are the horizontal and vertical deflections Δx and Δy . There can be little doubt that e_r is influenced by this variation in shape. However, since it was demonstrated that e_r could be evaluated for a given set of soil characteristics in terms of a constant, C_1 , it appears that the effect of variation of the pipe shape on e_r (pipe shape factor) is already included in C_1 . See Equation 19. The fact that C_1 was found to be essentially constant for any given set of soil characteristics may be due to the modifying influence of this pipe shape factor. Reasonably it would seem that since C_1 decreases as the settlement ratio increases negatively (see Figure 14), and since the settlement ratio increases negatively as the pipe

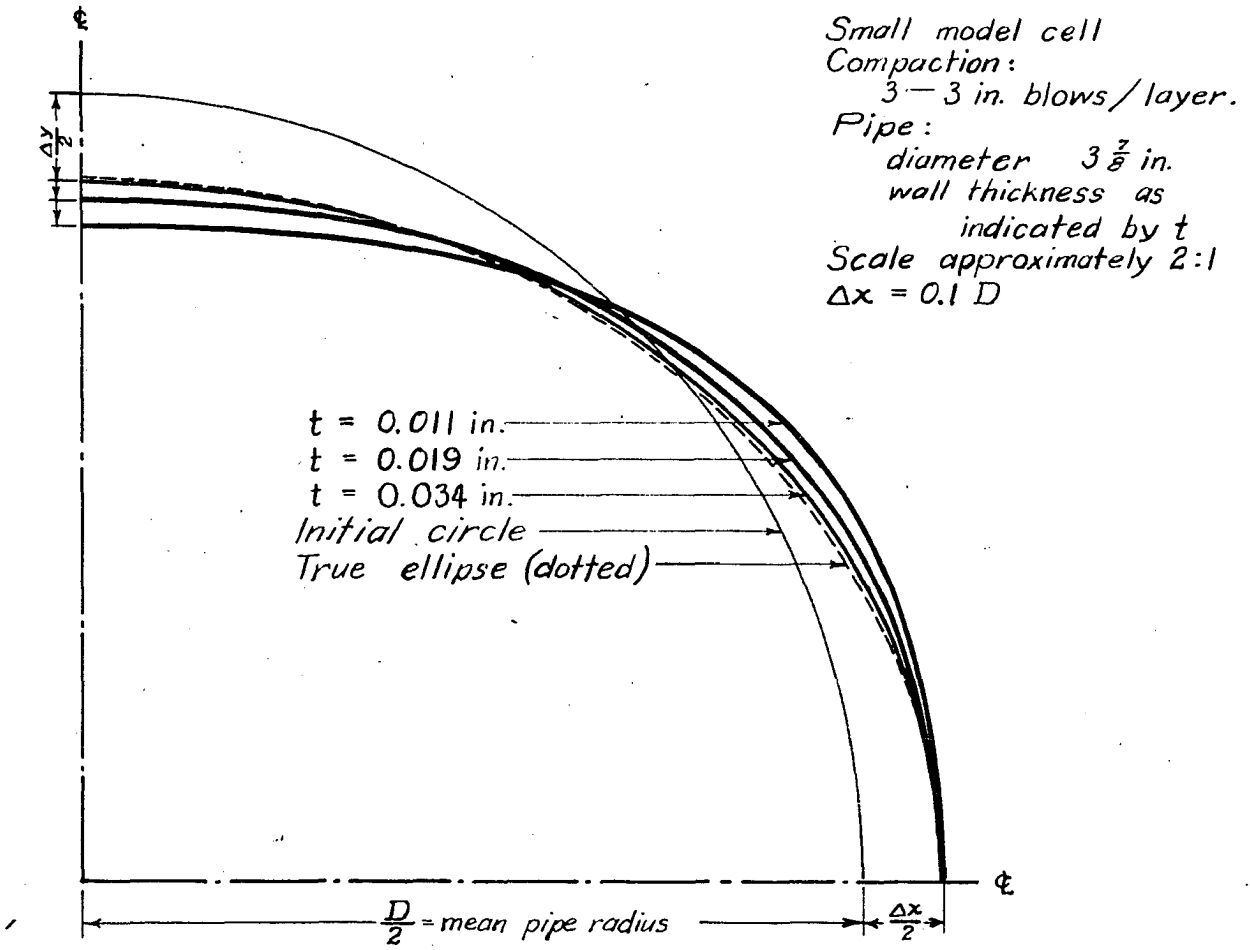


Figure 41. Comparison of pipe cross sections in loess at 10 per cent increase in horizontal diameter as a function of pipe wall stiffness.

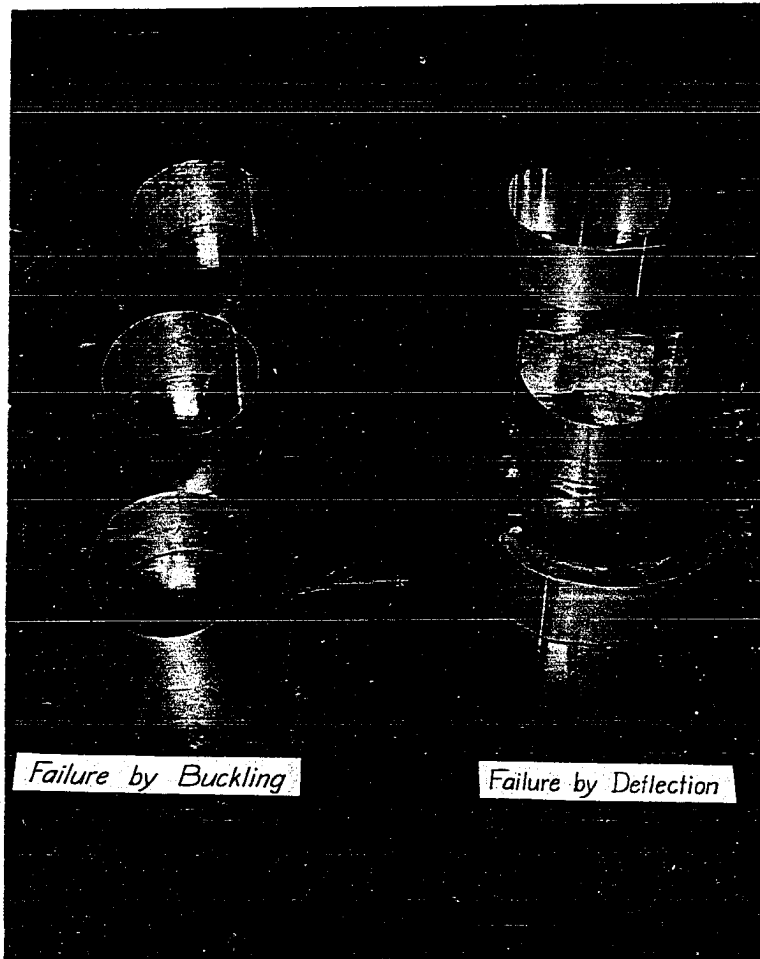
becomes more flexible, C_1 should decrease as the pipe becomes more flexible. On the other hand it would appear from Figure 41 that the flexible pipe adjusts in shape to accommodate greater vertical soil loads, so C_1 would tend to increase as the pipe becomes more flexible. A similar modifying effect of the pipe shape factor results from the influence of height of fill on C_1 . From Figure 36 it appears that the vertical compression of soil varies as the logarithm of the vertical soil load; but Figures 34 and 35 show that vertical deflection of the top of the pipe varies nearly linearly with respect to vertical soil load. It follows that the settlement ratio should increase negatively and C_1 decrease as the height of fill increases. On the other hand, the influence of the pipe shape factor increases as the pipe deflects, so again the pipe shape adjustment would tend to raise the value of C_1 as the height of fill increases. It may well be that the surprisingly large value of $C_1 = 3.2$ evaluated for loess is larger than the predicted values of 0.7 to 1.0 because of this shape factor. Judging from the shape of the lightest section in Figure 41 it is apparent that the horizontal deflection of the pipe wall is less than it would be if the elliptical shape prevailed. This means that the slope of the load-deflection curve for the lighter section is steeper than would be expected, and the value for C_1 is therefore greater than would be expected. See Table 3.

This may account for a value of C_1 as high as 3.2.

It has been shown in "Influence of Pipe Wall Stiffness on the Modulus of Soil Reaction" that C_1 may be taken as a constant for any soil at a given compaction, so the influence of the pipe shape needs no further attention. From these observations it also appears that the assumptions of constant C_1 and elliptical pipe cross section in the use of the Iowa Formula are justified since any small variations in the two "constants" would tend to counterbalance each other.

It is appropriate here to propose that further model study be directed toward the investigation of failure conditions. The comparison of pipe shapes reveals that pipe collapse is more probable at a given Δx in the case of the more flexible pipe because its shape is closer to that of reversed curvature. This is proven by the X-ray tests wherein pipe collapse occurred twice with the flexible section at a deflection of about 12 per cent. Equivalent deflections were observed with the stiffer sections, but at no time did the pipe collapse. Such an investigation of failure conditions could be carried out very satisfactorily by model study. Some typical pipe failures are shown in Figure 42. Pipe collapse is shown in the three pipe sections on the right labeled "Failure by Deflection". The center pipe section, L-2, assumed the odd shape because a dial gage was inside of it when it collapsed. All three of these failures

Figure 42. Typical failures of model pipe sections



Failure by Buckling

Failure by Deflection

occurred in loess at three 3-inch blows per layer and with a pipe wall thickness of 0.011 inch. The innertube pressure was between 145 psi. and 150 psi. at failure in all three cases. The three pipe sections on the left all buckled in sand at no compaction. The innertube pressures were 150, 155, and 160 psi. at failure. It is important to point out that buckling occurred in each case at the seam which was placed at 45 degrees from the top of the pipe section.

E. Experimental Verification of Prediction Equation

In order to investigate the characteristics of the modulus of passive resistance of soil, a prediction equation was derived whereby the modulus of passive resistance for a full scale field installation could be investigated by a model. See "Theory of Models as Applied to Flexible Pipe Culverts Under Earth Fills". All of the preceding results of this section have been determined expressly for a model. Now the question arises as to whether these same results hold for a prototype according to the prediction equation

$$e_r = (e_r)_m . \quad \text{Eq. 11}$$

The subscript, m, refers to the model and the modulus of soil reaction, e_r , is defined according to Equation 19 as

$$e_r = 1.36 C_1 D T S - 130 \frac{EI}{D^3} \quad \text{Eq. 19}$$

Equation 11 may now be rewritten in terms of Equation 19 as follows:

$$1.36 C_1 D T S - 130 \frac{EI}{D^3} = 1.36 C_{1m} D_m T_m S_m - 130 \frac{(EI)_m}{D_m^3} .$$

Eq. 20

Verification of the prediction equation means simply verification of Equation 20 provided the design conditions have been met. From "Theory of Models as Applied to Flexible Pipe Culverts Under Earth Fill," design conditions were established as follows:

1. All dimensions in the soil must be geometrically similar in model and prototype. The scale factor is $n = \frac{D}{D_m}$ where D is mean diameter of the pipe.

$$2. (EI)_m = \frac{EI}{n^4} .$$

3. All soil characteristics (including soil pressures) must be the same in the model and prototype.

The first design condition was met by constructing two models, one with all linear dimensions just half as large as the other. See Figures 11 and 13. Thus n is 2. The large model was considered to be the prototype.

The second design condition was met in the design of the model pipe. Both pipe sections were made out of sheet iron so that $E_m = E$. The ratio of the thickness of metal, t , was easily calculated to be

$$t_m = 0.396t.$$

The prototype pipe section was made out of 18 gage iron which was 0.050 inch thick and the model was 0.019 inch thick. Of course the diameter of the larger was twice the smaller.

The third design condition was met by using the same soil in each cell, compacted in layers of the same thickness and with four times as many blows per layer in the prototype cell.

Taking the above design conditions into account, Equation 20 becomes

$$2 C_1 T S = C_{1m} T_m S_m . \quad \text{Eq. 21}$$

It was shown in "Influence of Height of Fill on Modulus of Soil Reaction" that C_1 is a constant for any given pipe-fill system depending on settlement ratio, projection ratio, and pipe shape factor. If ratio of relative displacements in the prototype and model is n , then C_1 must equal C_{1m} . For high fills Figure 43 indicates that such must be the case so $C_1 = C_{1m}$. It is unfortunate that Figure 43 shows that the ratio of deflections is 2 for high fills only. The timber walls for the prototype cell were not constructed to proper tolerances and fit loosely after the pipe had been installed. Very likely the ratio of deflections would have been 2 throughout the entire range of values had that misfortune not occurred. For equal per cent deflection of the pipes the pressures on the tops of the pipes

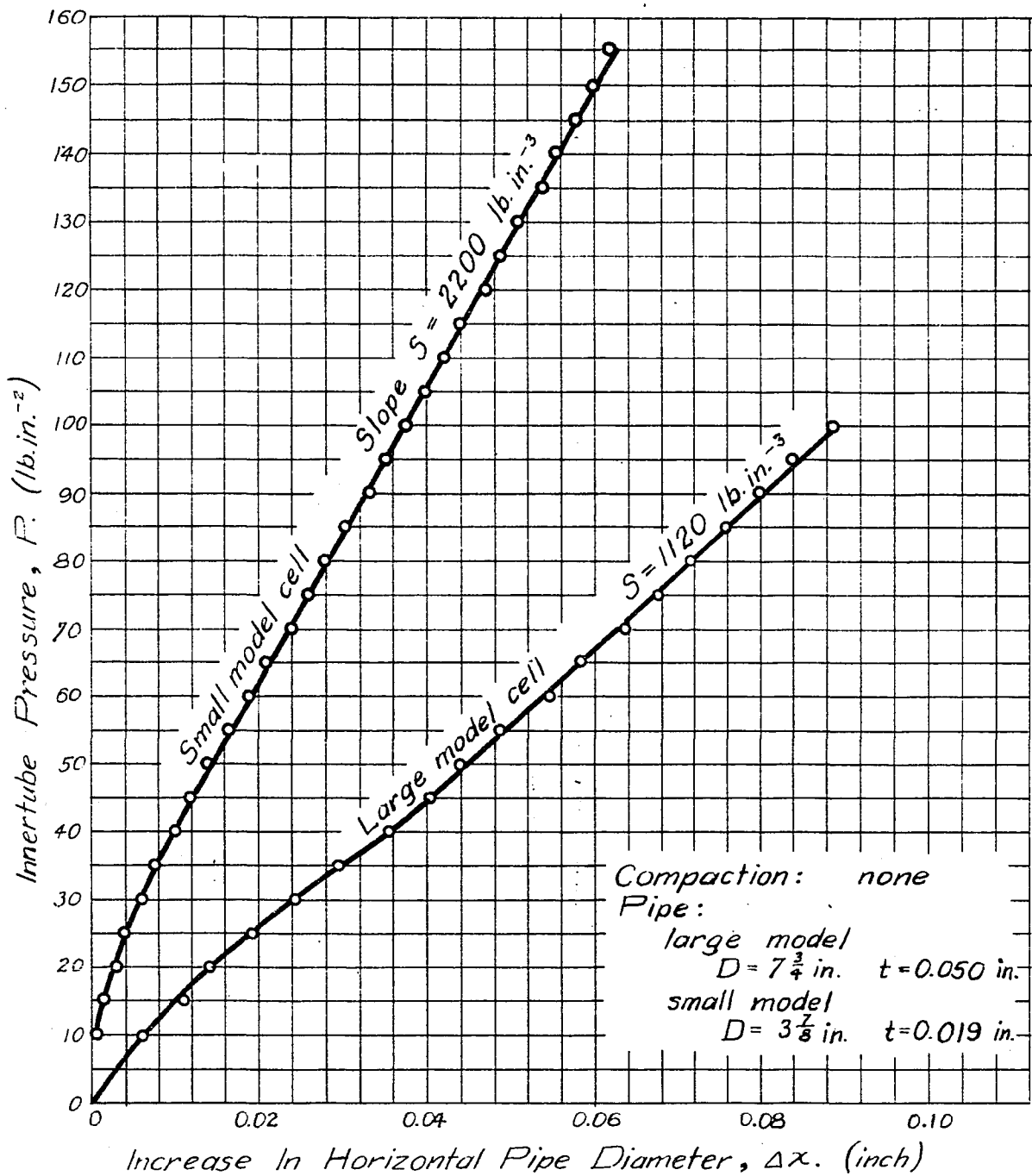


Figure 43. Comparison of load-deflection diagrams for two model systems for which all design conditions are satisfied

are the same, so $T = T_m$. Finally Figure 43 proves that $2S = S_m$ so Equation 21 is a true statement and the prediction equation is verified.

Replicability of results for the prototype cell as well as the model cell is easily established. For example, Figure 44 shows load-deflection diagrams for two slightly different pipe sections in uncompacted Saint Peter sand in the prototype cell. The upper diagram is a replot of Figure 39. It is easily shown that e_r is the same for both cases.

The important conclusion from the foregoing discussion is that the modulus of soil reaction, e_r , is independent of the size of the system, so e_r rather than e should be used as the basic soil modulus in applying the Iowa Formula. This fact generally clears up one point of confusion in the design of flexible pipe culverts. Using the Iowa Formula and assuming $e = 20 \text{ lb. in.}^{-3}$ to be the basic soil modulus, Kelley (7, p. 364) plotted values of height of fill, H , in feet as a function of diameter of the pipe, D , in inches. The plot is reproduced as Curve A in Figure 45. To the left of $D = 36$ inches the height of fill decreases as the diameter increases. This is to be expected. But to the right of $D = 36$ inches the height of fill increases. This defies intuitive judgment and is the source of considerable doubt regarding the validity of the Iowa Formula. If e_r , rather than e , is accepted as the basic soil modulus, and

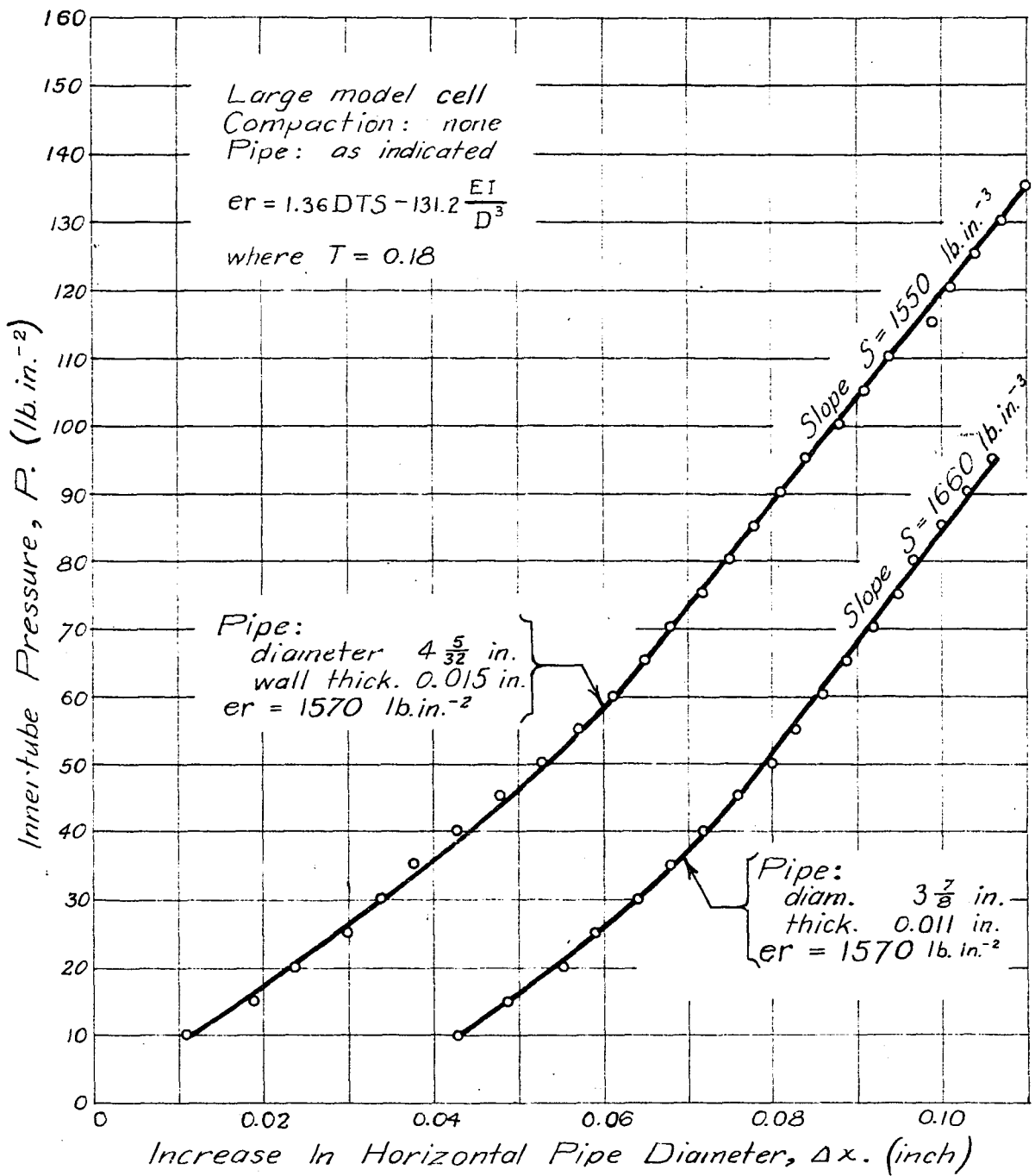


Figure 44. Load-deflection diagrams for two near duplicate tests on white silica sand (Saint Peter Formation) in the large model cell

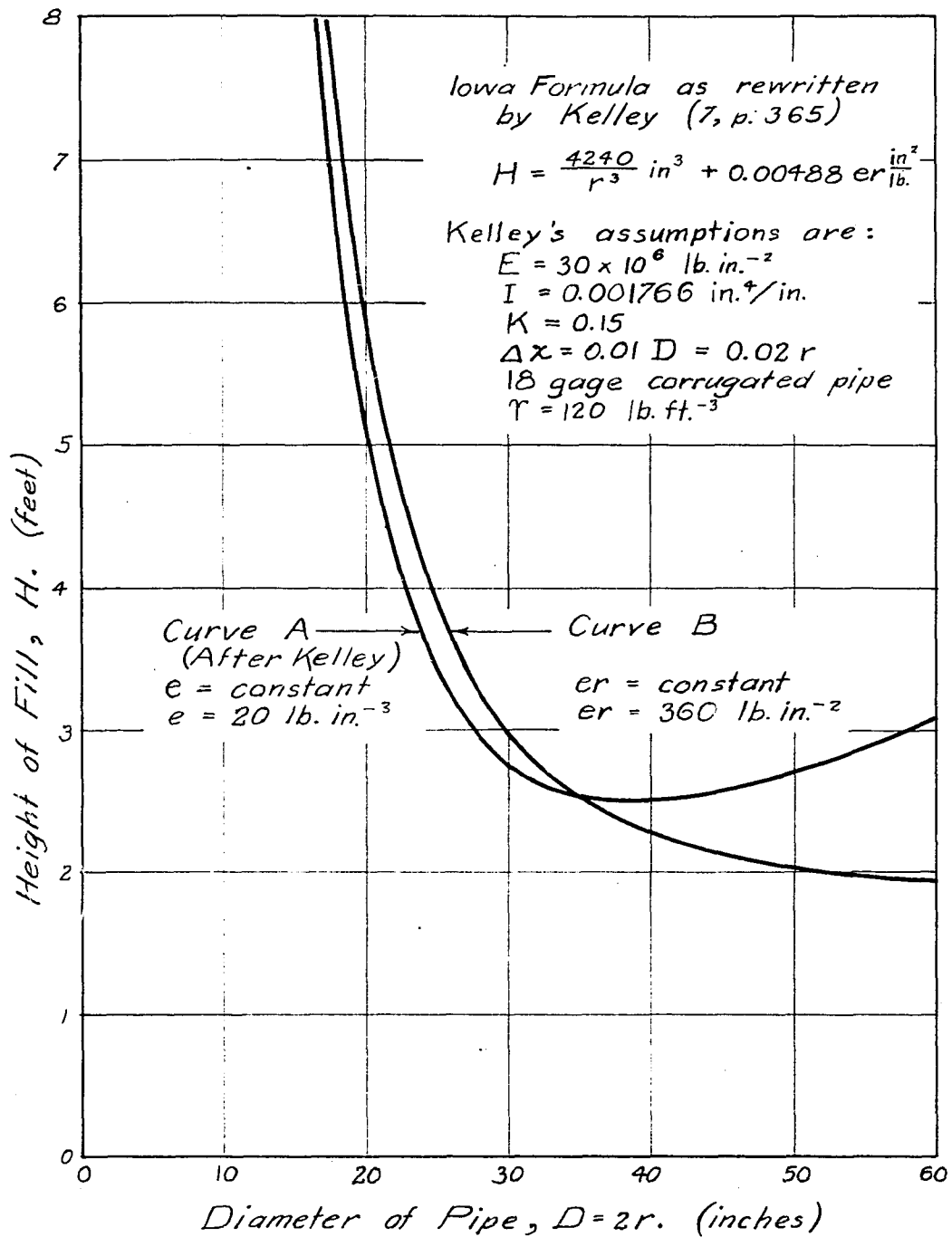


Figure 45. Comparison of plots of height of fill as a function of pipe diameter for er constant and for e constant according to Kelley (7, p. 364)

assuming that $er = 360 \text{ lb. in.}^{-2}$, Curve B results. Certainly Curve B appears more logical and lends confidence to the conclusions of this section.

Since er is independent of the size of the system it may be evaluated by means of a model.

F. Modification of the Spangler Theory

Spangler's theory as written in the form of Equation 15 must take on an additive constant if it is to account for the P-intercepts. From Equation 17 the required constant is $\frac{TP_0}{\gamma}$. With this additional term Equation 15 becomes

$$H = (er + 131.2 \frac{EI}{D^3}) \frac{\Delta x}{1.36 C_1 \gamma D} + \frac{TP_0}{\gamma} . \quad \text{Eq. 22}$$

The last term may be considered as an effective height of fill, H_0 , which is sustained by the interlocking of the soil particles. General design practice has established the limiting value of horizontal deflection as $\Delta x = 0.05 D$. Substituting these values in Equation 22 and multiplying through by γ ,

$$H\gamma = \frac{1}{C_1} \left[0.00368(er) + 0.483 \frac{EI}{D^3} \right] + H_0\gamma , \quad \text{Eq. 23}$$

where $H\gamma$ is the allowable vertical soil pressure above the level of the top of the pipe. A discussion of the term, H_0 , follows.

This project does not include evaluation of H_o but an inspection of Figures 22, 23, and 24 would indicate that TP_o or H_o , like er , is a function of the degree of compaction, Ω . It may logically follow that design of height of fill over flexible pipe culvert might take the form:

$$Hr = \frac{1}{C_1} \left[f_s(\Omega) + 0.48 \frac{EI}{D^3} \right], \quad \text{Eq. 24}$$

where the first term in the brackets is a constant depending on soil characteristics (including compaction), the second term is the contribution due to pipe strength only, and C_1 is the interrelationship between soil displacement and pipe deformation. Model studies would be required to evaluate the constants C_1 and $f_s(\Omega)$ for basic soil types. It must be emphasized that the modified Spangler theory as written in Equation 24 includes the original theory in that if H_o is zero, or if it may be assumed zero, Equation 24 is the original Spangler theory. Furthermore, for most flexible pipe culvert design, H_o may be conservatively set at zero.

G. Application of the Spangler Theory to an Existing Flexible Pipe-fill Installation

Careful records of performance have been kept on a rather famous flexible pipe-fill installation referred to as North Carolina Project 8521 on Highway US 70 between Ridgecrest and

Old Fort in western North Carolina.¹ The fill height is about 170 feet. The soil around the pipe is classified as A-4 according to the Bureau of Public Roads classification system. Compaction is approximately 95 per cent Proctor density. The soil weighs 105 pounds per cubic foot. According to these conditions, the load-deflection diagram for clay at twelve 12-inch blows per layer (see Figure 24) should apply approximately. For this diagram, $S = 630 \text{ lb. in.}^{-3}$. Using a boundary factor of 1.33 this slope would be 840 lb. in.^{-3} for a field installation. Since the silt-clay mixture is rather dense, C_1 is probably about unity, so $er = 880 \text{ lb. in.}^{-2}$ as calculated by methods of Table 3. The pipe of the North Carolina project was principally Armco Multi-plate pipe with 2 inch x 6 inch corrugations for which per inch of pipe length $I = 0.1458 \text{ in.}^3$. The diameter of the pipe was 66 inches, so $131.2 \frac{EI}{D^3} = 1930 \text{ lb. in.}^{-2}$. The maximum deflection of the pipe was measured to be 5.82 per cent. Now from Equation 25, $H = 163$ feet. This checks surprisingly well with the actual height of fill of 170 feet. The result is conservative as might be expected since H_0 was neglected.

¹Proudley, Charles E., Chief Materials Engineer, North Carolina State Highway and Public Works Commission, Raleigh, North Carolina. Field notes on Project 8521. Private communication, 11 January 1957.

VII. SUMMARY AND CONCLUSIONS

A brief summary of results is presented in this section together with some of the more important conclusions. The first results to be considered are those which bear directly on the objective of this project which was to investigate the modulus of passive resistance of soil and to establish a practical method of evaluating it. The designation of modulus of passive resistance is e . The radius of the pipe is r .

1. The quantity, er , rather than e is the basic soil modulus which should be used in the Iowa Formula. Model studies show that er is a property of soil characteristics only. In this respect it represents the same quality in the soil of a pipe-fill system that Modulus of Elasticity does in an elastic system. This statement is confirmed by the fact that the dimensions are the same. It is on this basis that er is referred to in this project as the modulus of soil reaction.

2. The modulus of soil reaction, er , is independent of the size of the pipe-fill system so it may be evaluated by a model study. The Iowa Formula can be rewritten to suit the conditions of a specific model, then it may be resolved for

er as follows:

$$e_r = 1.36 C_1 D T S - 131.2 \frac{EI}{D^3} . \quad \text{Eq. 19}$$

T is the transmission ratio for the model cell. It depends on the particular model cell used, and is a constant for a given soil. S is the slope of the load-deflection diagram for the model. D is the diameter of the pipe section. C_1 is a constant for any given set of soil characteristics since it was shown to be essentially independent of height of fill, pipe radius, and pipe stiffness. This investigation was limited to flexible pipe; that is, the pipe must deform according to the imperfect ditch condition. C_1 is a constant for model and prototype. It primarily represents the combined effects of settlement ratio, projection ratio, and pipe shape factor for the system. Equation 19 requires the evaluation of C_1 as well as e_r should the accuracy of design or the soil type demand that C_1 be different from unity. This may be done by plotting load-deflection diagrams for two or more model pipe-fill systems in which pipe wall stiffness, EI, is the only quantity varied. Two equations are then available for evaluating e_r and C_1 . For clean, granular material C_1 may be considered unity, and e_r may be evaluated for any given degree of compaction by a single test.

3. The modulus of soil reaction, e_r , is independent of

height of fill, H , in the range for which the load-deflection diagram is a straight line. The upper limit of linearity for all load-deflection diagrams of this project is at least 7 per cent horizontal deflection. Above this deflection there are breaks in the diagrams. The lower portions of all load-deflection diagrams are curved depending on the variation of the degree of compaction from critical. For degree of compaction greater than critical, the lower limit of linearity is at most 2 per cent deflection. It follows, then, that e_r is useful between about 2 per cent and 7 per cent horizontal deflection, but since this range includes the 5 per cent deflection commonly used for design, it is considered adequate.

4. The modulus of soil reaction, e_r , is independent of pipe wall stiffness, EI . This presumes that the proper value of C_1 has been determined.

5. Unless all boundaries are geometrically similar and otherwise the same in both model and prototype the walls of the model cell must be spaced sufficiently far apart to include boundaries of relative soil displacement in the soil. The top boundary of the model must be high enough to include the plane of equal settlement at maximum pipe deflection. This condition establishes a minimum height of fill for which the model cell and prototype obey principles of similitude. Consequently the minimum height of fill for which this type

of model cell applies is the height to the plane of equal settlement under maximum deflection conditions (usually specified to be 5 per cent).

The following results of this project do not bear directly on the characteristics or evaluation of e_r , but since they apply to the application of e_r in the Iowa Formula their inclusion here seems justified.

6. Since both the modulus of soil reaction, e_r , and the constant, C_1 , can be evaluated for a model and since neither expression varies for model or prototype, the height of fill, H , for a field installation may be calculated by means of the Iowa Formula if modified and rewritten as follows:

$$H\gamma = \frac{1}{C_1} \left(e_r + 131.2 \frac{EI}{D^3} \right) \frac{\Delta x}{1.36D} + H_0\gamma. \quad \text{Eq. 25}$$

$H\gamma$ is the allowable soil pressure at the level of the top of the pipe due to height of fill, H , and unit weight of the soil, γ . $H_0\gamma$ is a constant depending on the soil characteristics and particularly on the degree of compaction. Ranges of values can easily be determined by model studies since $H_0\gamma = TP_0$ where P_0 is the P-intercept of the load-deflection diagram and T is transmission ratio for the particular model cell and soil type. For most design work under present methods of inspection and control, it is sufficiently accurate on the conservative side to neglect

the term, H_0 . If H_0 is assumed to be zero the degree of compaction of the fill in the proximity of the pipe must be specified to be greater than critical compaction. The quantity, Δx , is usually specified as the design condition for a flexible pipe culvert. Ordinarily it is set at 5 per cent of the diameter, D . Under this design condition the quantity, $\frac{\Delta x}{1.36 D}$, becomes 0.037. The term, $131.2 \frac{EI}{D^3}$, is the contribution due to pipe stiffness. It is easily evaluated for any pipe.

If H_0 is zero or may be assumed zero, Equation 25 is the original Spangler theory. By far the greater majority of flexible pipe design problems will fall in this category. An evaluation of H_0 could only be justified on very costly projects wherein the conservatism of the Spangler theory is not economical.

7. There is evidence that a constant pipe deflection is not the best criterion for design. At a given deflection such as $\Delta x = 12$ per cent of pipe diameter, very flexible pipes will collapse whereas stiff pipes will not. More investigation is needed at or near conditions of pipe collapse to determine just what per cent deflection really defines failure.

8. Model study is not limited to the evaluation of e_r . Design conditions may be conveniently met which make possible

the direct prediction of deflection of a flexible pipe by use of a model. Such a procedure would probably be confined to high cost installations because of the cost of performing model studies. When even greater accuracy can be justified in the design of flexible pipe culverts, models can also be used to investigate the effect of stresses in the third dimension or in the direction of the pipe axis.

9. Failure of flexible pipe by buckling rather than by deflection looms as a serious problem as the design and control of pipe-fill installations is improved. There is need for study in this area to determine lower limits of pipe-wall stiffness above which the Spangler theory is valid.

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Charles E. Proudley, Chief Materials Engineer, and C. R. Penny, Materials Engineer, both of the North Carolina State

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Highway and Public Works Commission were most courteous in providing information on the famous flexible pipe culverts projects on Highway U.S. 70 between Ridgecrest and Old Fort in western North Carolina.

X. APPENDIX

A. Angle of Passive Bearing Surface of Flexible Pipe Culvert

When a flexible pipe deflects under an earth fill the horizontal diameter increases and develops passive or partially passive soil pressures against the sides of the pipe. See Figure 46. In this project it is necessary to estimate the angle of pipe-soil contact where passive pressure tends to develop. A logical approach is to assume that the circumference of the pipe remains constant during deformation and that the cross section remains elliptical. Under these circumstances the points of intersection of the ellipse with its initial circular position can be located. The angle of passive bearing surface, 2θ , can then be determined. See Figure 47. The equation of an ellipse is:

$$b^2x^2 + a^2y^2 = a^2b^2.$$

The equation of a circle is

$$x^2 + y^2 = r^2.$$

Now to relate a and b to r , for very small deflections for which the circumference remains constant,

$$\frac{a + b}{2} \approx r.$$

This is based on the common assumption that the decrease in vertical diameter equals the increase in horizontal diameter.

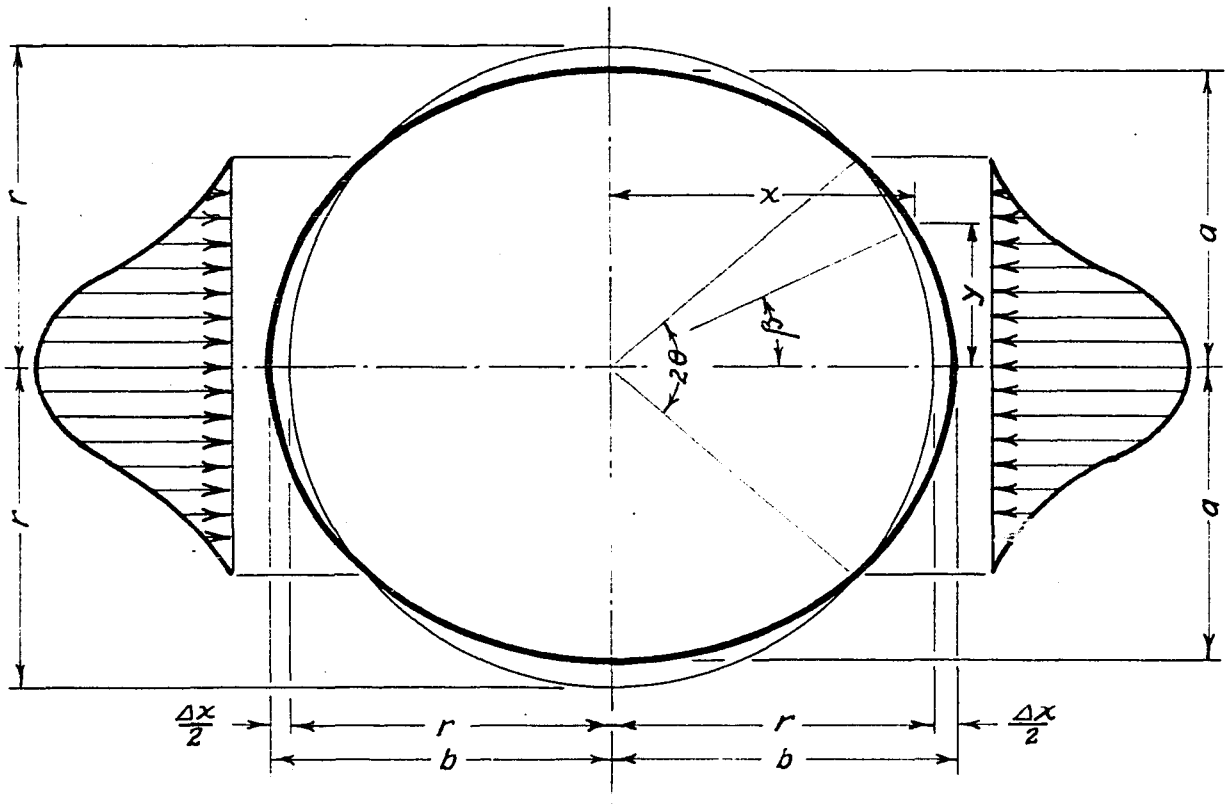
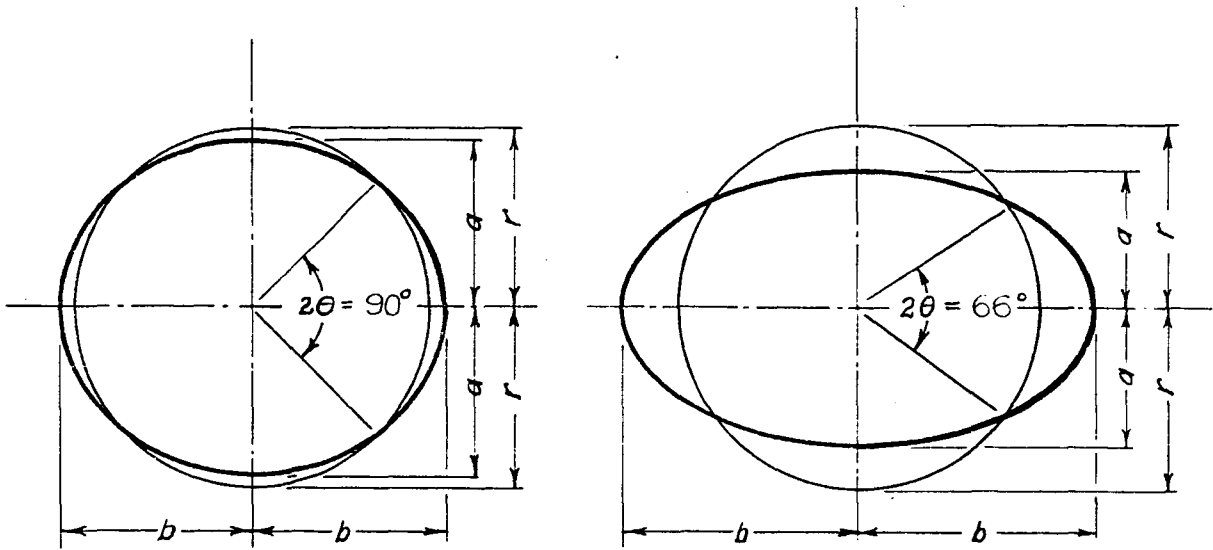


Figure 46. Angle of passive bearing surface, 2θ , of flexible pipe culvert against laterally adjacent soil



(a) $2\theta = 90^\circ$
 as $b \rightarrow r$
 and $a \rightarrow r$

(b) $2\theta = 66^\circ$
 for $b = 1.20r$
 and $a = 0.78r$

Figure 47. Angles of passive bearing surface, 2θ , if b and a are assigned limiting values

It may be substituted into the equation of a circle, then the equations for a circle and an ellipse may be solved simultaneously for X and Y at the intersections. The results follow:

$$X = \frac{b^2 r^2 - a^2 b^2}{b^2 - a^2}$$

$$Y = \frac{a^2 r^2 - a^2 b^2}{a^2 - b^2}$$

Eq. 26

In these equations $b > a$. The angle of contact, 2θ , can be calculated from the relationship

$$\theta = \tan^{-1} \frac{Y}{X}.$$

Eq. 27

A few values are listed in Table 4.

The value of $\theta = 45^\circ$ for $a = b = r$ may be determined by evaluating the limit as a approaches b of the equations for X and Y. For example, letting $Y_r = \lim_{a \rightarrow b \rightarrow r} Y$,

$$Y_r^2 = \lim_{a \rightarrow b \rightarrow r} \left[\frac{b^2 r^2 - a^2 b^2}{b^2 - a^2} \right]$$

$$= \lim_{\Delta \rightarrow 0} \left[\frac{(r+\Delta)^2 r^2 - (r-\Delta)^2 (r+\Delta)^2}{(r+\Delta)^2 - (r-\Delta)^2} \right]$$

where $\Delta = b - a$. Expanding and collecting terms

$$Y_r^2 = \lim_{\Delta \rightarrow 0} \left[\frac{3r^2 \Delta^2 - \Delta^4 + 2r^3 \Delta}{4r \Delta} \right]$$

$$Y_r^2 = \lim_{\Delta \rightarrow 0} \left[\frac{3}{4} r \Delta - \frac{\Delta^3}{4r} + \frac{1}{2} r^2 \right]$$

Table 4. Coordinates of intersection of ellipse with concentric circle of equal circumference; and angle of passive bearing surface of flexible pipe against laterally adjacent soil by approximate solution

a	b	Coordinates of intersection		θ
		X	Y	
1.00r	1.00r	0.707r	0.707r	45.0°
0.99r	1.01r	0.713r	0.702r	44.6°
0.98r	1.02r	0.718r	0.697r	44.2°
0.97r	1.03r	0.723r	0.691r	43.8°
0.96r	1.04r	0.728r	0.685r	43.3°
0.95r	1.05r	0.733r	0.680r	42.9°
0.9 r	1.1 r	0.759r	0.653r	40.7°
0.8 r	1.2 r	0.805r	0.594r	36.5°

$$Y_r^2 = \frac{1}{2} r^2 \quad \text{or} \quad Y_r = \frac{r}{\sqrt{2}}$$

Likewise
$$X_r = \frac{r}{\sqrt{2}}$$

and
$$\theta = 45^\circ.$$

There is a slight error involved in the common assumption that $\frac{a+b}{2} = r$ if the circumference of the circle is to be the same as the circumference of the ellipse. This error can easily be pointed out by evaluating an equation for the circumference of an ellipse, s_e , and comparing it with the circumference of a circle, $s_c = 2\pi r$. The equation of an

ellipse may be written in the parametric form,

$$x = b \cos \beta \quad \text{or} \quad dx = -b \sin \beta \, d\beta$$

$$y = a \sin \beta \quad \text{or} \quad dy = a \cos \beta \, d\beta$$

where β is as indicated on Figure 46. Now since $ds_e = \sqrt{dx^2 + dy^2}$,

$$ds_e = \sqrt{b^2 \sin^2 \beta + a^2 \cos^2 \beta} \, d\beta$$

$$ds_e = \sqrt{1 - k^2 \sin^2 \beta} \quad \text{where} \quad k^2 = \frac{b^2 - a^2}{b^2} .$$

The expression for k^2 is known as the eccentricity of the ellipse. Integrating from $\beta = 0$ to $\beta = 2\pi$ (4, p. 504)

$$s_e = 2\pi b \left[1 - \left(\frac{1}{2}k\right)^2 - \frac{1}{3} \left(\frac{1 \cdot 3}{2 \cdot 4} k^2\right)^2 - \frac{1}{5} \left(\frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} k^3\right)^2 - \dots \right]$$

If b is assigned the value $b = 1.1r$

$$s_e = 6.29996 \, r .$$

But if

$$s_e = s_c ,$$

$$s_e = 2\pi r = 6.28318 \, r .$$

Working backwards from $s_e = 6.28318r$ it is found that a cannot be $0.9r$ but rather $a = 0.8947r$. See Figure 48a for further comparison of values. For $b = 1.1000r$ and $a = 0.8947r$, $\theta = 39.2^\circ$, instead of 40.7° as listed in Table 4. The resulting discrepancy is taken into account in the plots of Figure 48 labeled Accurate Values.

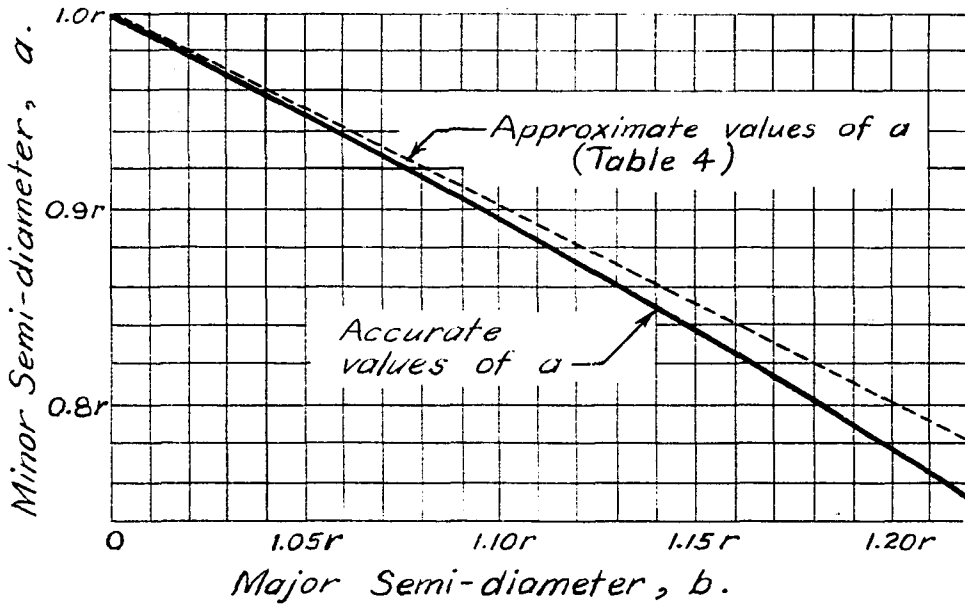


Figure 48a. Values of a as a function of b for an ellipse if the circumference remains constant

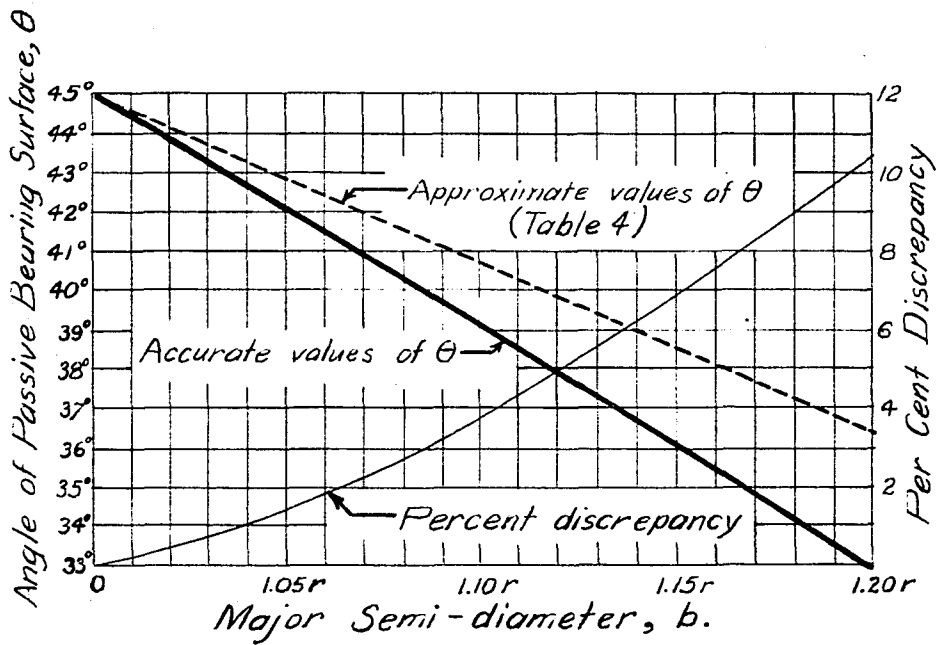


Figure 48b. Values of θ as a function of b showing the error in Table 4

B. Discussion of Shape of Inflated Membrane as Passive
Bearing Surface for Modpares Device

Figure 49a compares the shapes of the inflated membrane (which is a circle) and the assumed elliptical shape of the deflected pipe if $a = 0.9 r$ and $b = 1.1 r$. Figure 49b shows the difference between the deflections of the membrane and the ellipse if $a = 0.9 r$ and $b = 1.1 r$. The deviation is not enough to cause concern. In the first place the actual shape of the pipe varies from the assumed ellipse more than does the membrane. See Figure 41. In the second place, the shape of the bearing surface is only of secondary importance since the primary data to be observed are the maximum soil pressure, h , and the maximum deflection $\frac{\Delta x}{2}$. Thirdly, an unknown factor may be needed to convert air pressure on the membrane to maximum soil pressure. This factor will probably account also for the difference in the shapes of the membrane and the actual deflected pipe.

The logical conclusion is that a membrane may be substituted for the pipe wall in order to apply pressure to the soil, provided that some constant of proportionality be introduced to convert air pressure to maximum horizontal soil pressure.

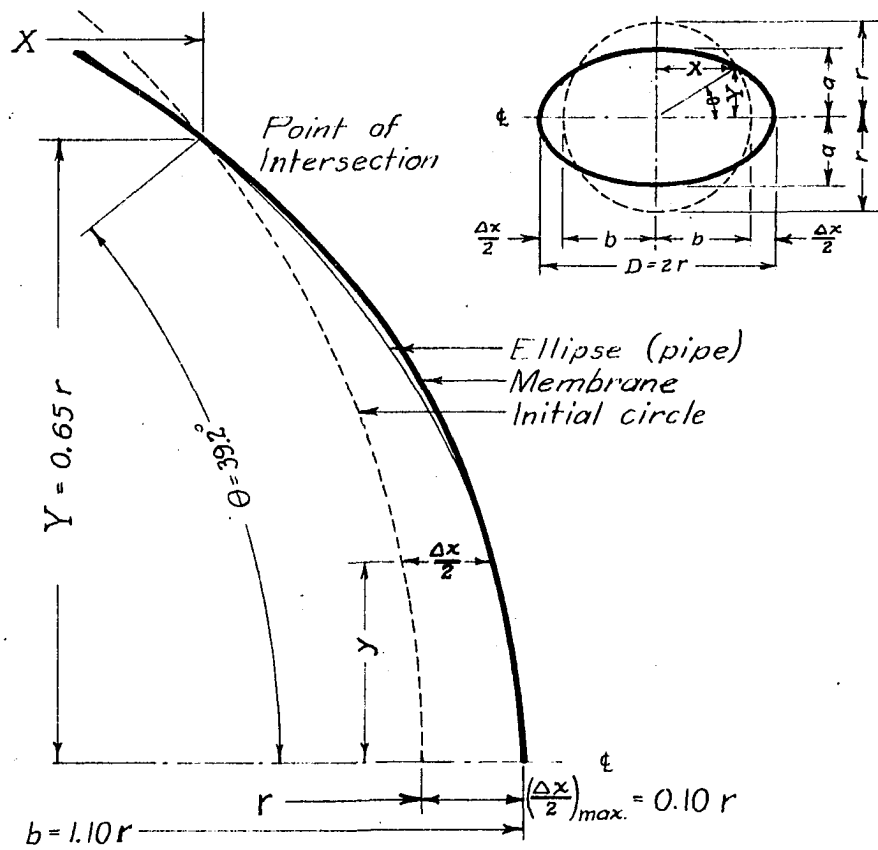


Figure 49a. Comparison of shapes of inflated membrane (circle) and deformed pipe (assumed to be an ellipse) if $b = 1.1r$ and $a = 0.9r$

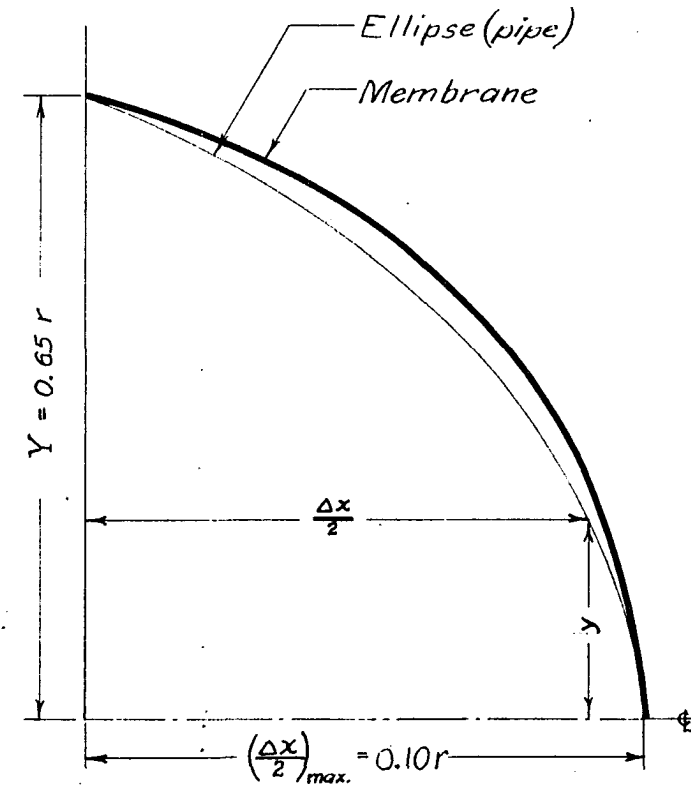


Figure 49b. Comparison of deflections, Δx , of inflated membrane and deformed elliptical pipe

C. Rational Demonstration of Prediction Equation

A rational derivation of the prediction equation, $e = \frac{e_m}{n}$, is contained in this Appendix. Let the two parallelopipeds in Figure 50a represent pressure cylinders (pressure bulbs) adjacent to two pipes of different size. The smaller is assumed to be a model of the larger. The term, pressure cylinder (pressure bulb) is here defined as that surface on which the horizontal component of direct stress, p_x , is the same at every point; say, for instance, $p_x = 0.1h$. Actually the pressure cylinders should look more like Figure 50c, but it is easier at the moment to visualize the deformation of parallelopipeds as in Figure 50a. The conclusions are the same for either figure. These parallelopipeds are dimensionally similar according to the Design Conditions. The ratio of linear dimensions is the length scale factor; that is, $n = \frac{L}{L_m}$.

Now the two parallelopipeds may be thought of as short compression members lying on their sides as seen in free body diagrams of pressure cylinders in Figure 50b. Assume that the unit stress, h , is the same on the ends of both short columns. This is accomplished if the soil characteristics (particularly pressures in this case) are the same in model and prototype. Now assume that the columns decrease in length by $\frac{x}{2}$, then if Hooke's Law may be assumed to apply,

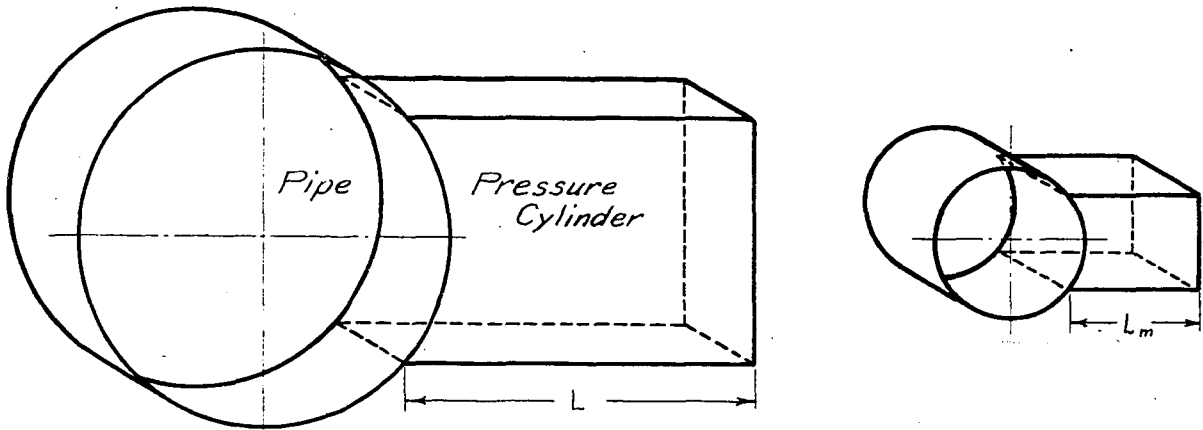


Figure 50a. Idealized pressure cylinders adjacent to flexible pipe

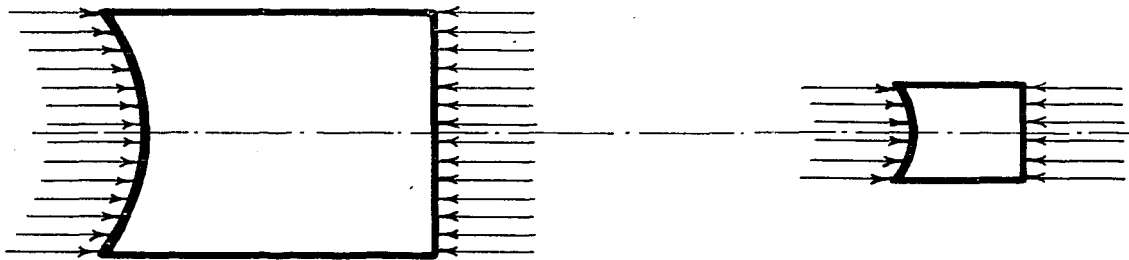


Figure 50b. Free body diagram of idealized pressure cylinders

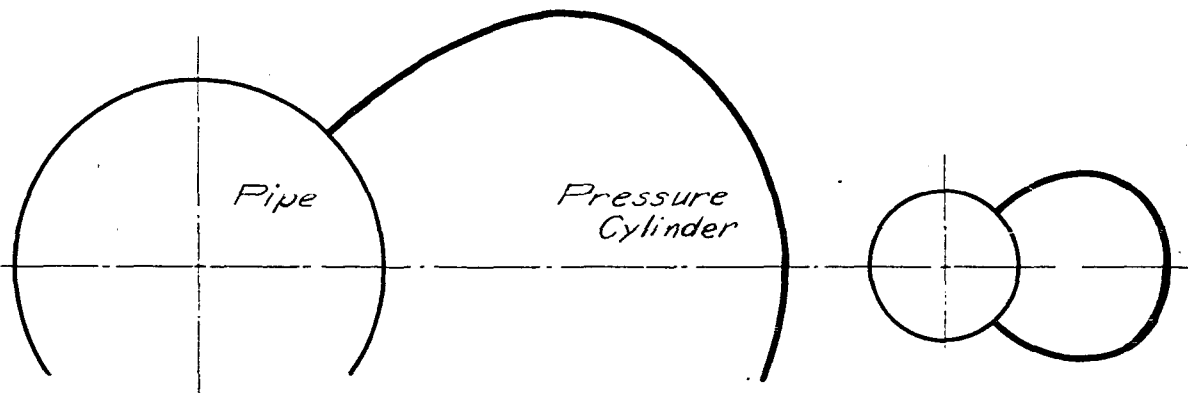


Figure 50c. Probable shape of pressure cylinders

Prototype

$$\frac{\Delta x}{2} = h \frac{L}{E}$$

Model

$$\frac{\Delta x_m}{2} = h_m \frac{L_m}{E_m}$$

where E = modulus of elasticity. Rewriting

$$\Delta x = 2h \frac{L}{E}$$

$$\Delta x_m = 2h_m \frac{L_m}{E_m}$$

or
$$h = \frac{E \Delta x}{2L}$$

$$h_m = \frac{E_m \Delta x_m}{2L_m} .$$

Now since $e = \frac{2 d(h)}{d(\Delta x)}$ by definition, then

$$e = \frac{E}{L}$$

and

$$e_m = \frac{E_m}{L_m} .$$

Again if soil characteristics are the same, $E = E_m$; and if by similarity $L = nL_m$, then

$$e = \frac{e_m}{n} .$$

This confirms the prediction equation.

There are other ways of arriving at the same conclusion using rational methods, but all of them require the same basic assumptions; i.e., the soil is an elastic material and the proportional limit is not exceeded. Obviously these assumptions are not true, so the above demonstration is questionable. Still soil action and elastic action are of such similarity that the above result is of interest.

D. Proof of Design Conditions by General Equations
of Elasticity

A check on the design conditions follows immediately from the general equations of elasticity which theoretically make possible the solution of two dimensional stress. The equations according to Timoshenko and Goodier (25) follow: (Notation in the Appendix follows Timoshenko rather than notation adopted for the report in general.)

Equations	$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + X = 0$
of	
Equilibrium	$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + Y = 0$

Boundary	$\bar{X} = l \sigma_x + m \tau_{xy}$
Conditions	$\bar{Y} = m \sigma_y + l \tau_{xy}$

Combined Compatibility Equation after Timoshenko and Goodier (25, p. 25) (Hooke's Law applies.)

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) (\sigma_x + \sigma_y) = \frac{1}{1-\nu} \left(\frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} \right)$$

σ_x and σ_y are the x and y components of direct stress.

τ_{xy} is shear stress on the x and y planes (planes perpendicular to the x and y axes).

X and Y are x and y components of body force.

(Body force is defined as any force acting throughout the volume of a body in contradistinction to an external force which acts on the surface. Examples of body forces are gravity, magnetic attraction or repulsion, and inertia if the body is accelerating.)

\bar{X} and \bar{Y} are the x and y components of surface forces or surface tractions.

l and m are x and y direction cosines of the normal to the boundary.

ν is Poisson's ratio.

In the above equations, the stress is uniquely determined at any location provided the body force, the boundary conditions and Poisson's ratio are defined throughout.

Suppose now that the same equations are applied to a model whose x and y length scales are $x_m = \frac{x}{n}$; and $y_m = \frac{y}{n}$; and whose stress scale is unity (i.e., $\sigma_x = \sigma_{x_m}$ and $\tau_{xy} = \tau_{xy_m}$). One equation of equilibrium becomes:

$$\frac{\partial \sigma_{x_m}}{\partial x_m} + \frac{\partial \tau_{xy_m}}{\partial y_m} + X_m = 0$$

where
$$\frac{\partial \sigma_{x_m}}{\partial x_m} = \frac{\partial \sigma_x}{\partial x} \frac{dx}{dx_m} = n \frac{\partial \sigma_x}{\partial x}$$

and
$$\frac{\partial \tau_{xy_m}}{\partial y_m} = \frac{\partial \tau_{xy}}{\partial y} \frac{dy}{dy_m} = n \frac{\partial \tau_{xy}}{\partial y}$$

or
$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{1}{n} X_m = 0 .$$

But from the original equation of equilibrium,

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + X = 0$$

so $X_m = nX$.

By similar reasoning $Y_m = nY$.

Again it is clear that the body forces must be n-times as heavy in the model; that is, the unit weight of the soil must be n-times as great in the model as in the prototype. This discussion partially confirms the third statement under Design Conditions II.

As far as boundary stresses are concerned

$$\bar{X}_m = \bar{X}$$

and

$$\bar{Y}_m = \bar{Y}$$

from the equations of boundary conditions so long as

$$\sigma_{x_m} = \sigma_x$$

and

$$\tau_{xy_m} = \tau_{xy}$$

Of course, $\lambda_m = \lambda$ and $m_m = m$ from geometrical similarity.

The above conditions indicate that the equations which establish boundary conditions are satisfied if all stresses, internal and boundary, are the same in model and prototype.

Finally, Poisson's ratio, ν , may be investigated from the equation of compatibility. For the model this equation becomes

$$\left(\frac{\partial^2}{\partial x_m^2} + \frac{\partial^2}{\partial y_m^2} \right) (\sigma_{x_m} + \sigma_{y_m}) = - \left(\frac{1}{1-\nu} \right)_m \left(\frac{\partial X_m}{\partial x_m} + \frac{\partial Y_m}{\partial y_m} \right)$$

where the following relationships hold from the preceding paragraph:

$$\begin{aligned} X_m &= nX \\ Y_m &= nY \\ \sigma_{x_m} &= \sigma_x \\ \sigma_{y_m} &= \sigma_y \\ \tau_{xy_m} &= \tau_{xy}. \end{aligned}$$

Substituting in these values the second parenthesis on the right becomes

$$\begin{aligned} \left(\frac{\partial X_m}{\partial x_m} + \frac{\partial Y_m}{\partial y_m} \right) &= n \frac{\partial X}{\partial x} \frac{dx}{dx_m} + n \frac{\partial Y}{\partial y} \frac{dy}{dy_m} \\ &= n^2 \left(\frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} \right) \end{aligned}$$

Also
$$\frac{\partial \sigma_{x_m}}{\partial x_m} = \frac{\partial \sigma_x}{\partial x} \frac{dx}{dx_m} = n \frac{\partial \sigma_x}{\partial x}$$

and
$$\frac{\partial^2 \sigma_{x_m}}{\partial x_m^2} = n^2 \frac{\partial^2 \sigma_x}{\partial x^2}$$

Expanding the same reasoning,

$$\left(\frac{\partial^2}{\partial x_m^2} + \frac{\partial^2}{\partial y_m^2} \right) (\sigma_{x_m} + \sigma_{y_m}) = n^2 \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) (\sigma_x + \sigma_y)$$

Substituting equivalent prototype terms for the model terms in the equation of compatibility, n^2 cancels out and

$$\left(\frac{1}{1-\nu} \right)_m = \left(\frac{1}{1-\nu} \right) \cdot$$

This indicates that Poisson's ratio must be the same in both

model and prototype. The idea of Poisson's ratio in soils is certainly not exact, for soil is not an elastic material. Nevertheless Bousines, Westergaard, Newman, and others, according to Taylor (21, pp. 250-266), have found that stresses can be calculated in soils by use of theory of elasticity with generally acceptable accuracy, provided relative displacements are very small as in the case of stresses below a footing. But even if the relative displacements are large as around flexible culverts, soil does have a modulus of passive resistance which is comparable to the Modulus of Elasticity even though it may not remain constant as the Modulus of Elasticity is assumed to do in the theory of elasticity. By similar reasoning there is undoubtedly a phenomenon in soils similar to Poisson's ratio. Whatever this soil property may be, it would certainly follow that two soils in which all other characteristics are the same would have the same "Poisson's ratio". For lack of more knowledge about soils this suggests that all soil characteristics should be the same in both model and prototype to insure that Poisson's ratio be the same. This further confirms the third statement in the Design Conditions.

E. Spacing of X-Boundaries in Model According to
Peck's Bearing Theory

1. For sand (wet or dry)

Figure 51a shows a cross section of the model. The spacing of the X-boundaries is indicated by L_x . Peck's work on bearing capacity of sand for footings (11, pp. 219-233), would indicate that X-boundaries vary markedly as a function of the internal friction angle, ϕ , of the soil. If his theory is applied to culverts the X-boundary spacing varies with ϕ as shown in Figure 51c. These results are based on Peck's plot of $N_\gamma = k(N_\phi^2 - 1)$ which is reproduced here in Figure 51b. As demonstrated in Appendix A, it may be assumed that $B = 0.707 D$; so $L_x = 2kB + D = (1.414 k + D)$ where k may be calculated from the relationship $k = N / (N_\phi^2 - 1)$. N_ϕ is a function of friction angle, ϕ , only. It is defined as $N_\phi = \tan^2(45^\circ + \frac{\phi}{2})$ and represents the ratio of maximum and minimum principal stresses at failure (11, p. 87). As ϕ varies from 30° to 40° , L_x varies from approximately $2D$ to $9.5D$.

2. For clay (saturated)

Figures 52a and 52b show the general cross section of the model. The X-boundary spacing is indicated as L_x . The probable horizontal soil pressures developed by the displacement of the pipe into the soil would plot somewhat as

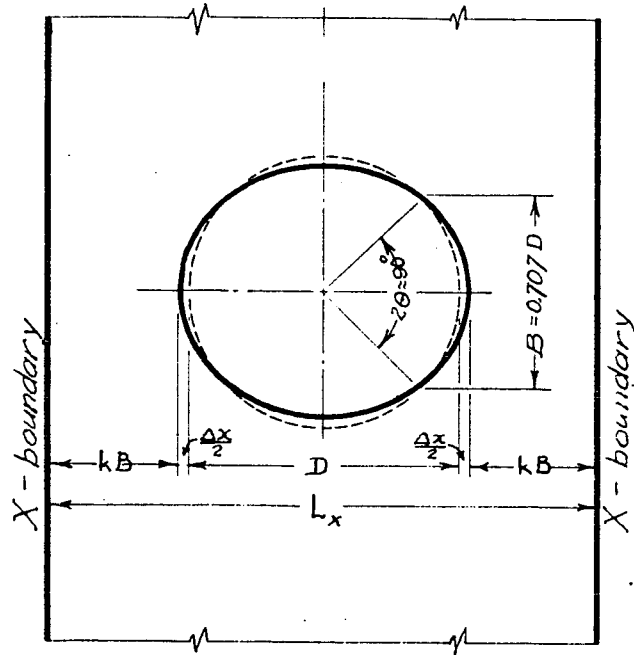


Figure 51a. Cross section of model cell showing dimensions

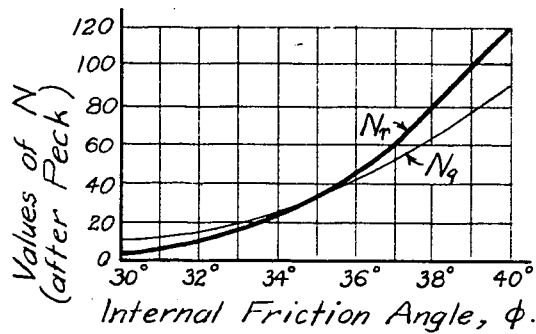


Figure 51b. Values of N_r and N_q as a function of friction angle, ϕ , for sand

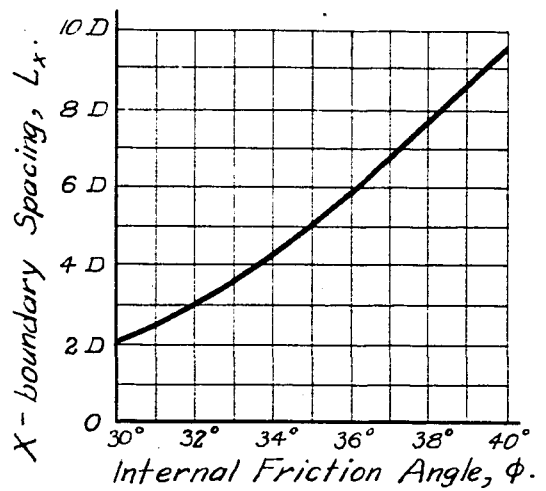


Figure 51c. X-boundary spacing in sand as a function of ϕ

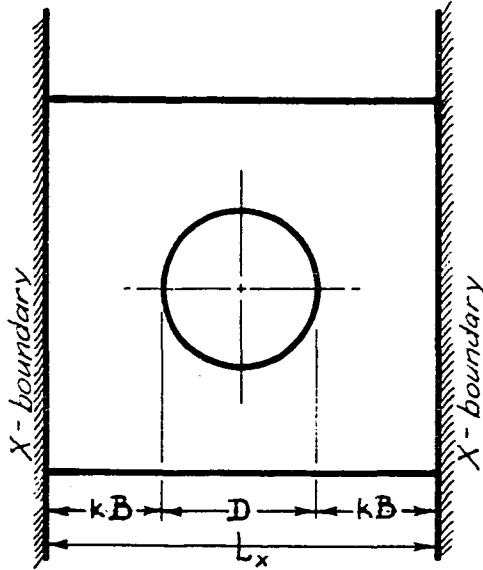


Figure 52a. Diagrammatic cross section of cell without load

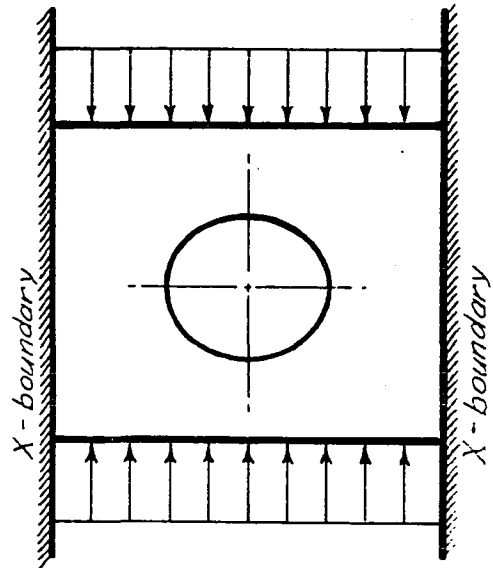


Figure 52b. Diagrammatic cross section of cell loaded

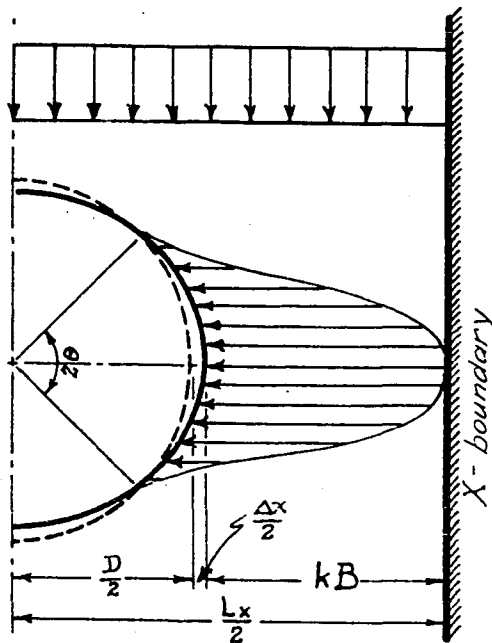


Figure 52c. Probable horizontal soil pressures developed by displacement of pipe in clay

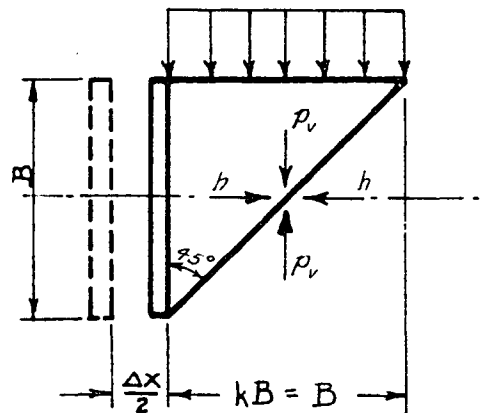


Figure 52d. Failure wedge caused by horizontal displacement of long, rigid rectangular plate

shown in Figure 52c; but in order to apply Peck's bearing method it must be assumed that the bearing surface is a long rectangle in shape and that it is perfectly rigid as shown in Figure 52d. In the case of saturated clay it is customary to neglect any friction angle and to assume that shear strength in the soil is derived from cohesion only. Under these assumptions failure will occur on a 45° angle plane as shown in Figure 52c (11, p. 250). Now if the width of the plate is B, it is apparent that the relative soil displacement will extend a distance B laterally into the soil; that is, on Figure 52c, $k = 1$.

Applying the same general theory to a culvert, it might be reasonable to assume that 90° of arc of pipe projects into the soil. See Figure 52c. (This is conservative, for as shown in Appendix A, $2\theta = 90^\circ$ just as the pipe starts to deflect, but by the time $\Delta x = 0.1D$, 2θ is only about 80° .) Based on this assumption, $B = D \sin \theta = 0.7D$, and the spacing becomes $L_x = 2B + D = 2.4D$; or conservatively the spacing of the X-boundaries is $L_x = 2.5D$.

3. For clay (unsaturated)

No attempt is made to predict the location of the X-boundaries for unsaturated clay since it will vary between wide limits. One limit is the same as for saturated clay; i.e., $L_x = 2.5D$. The other limit could be approximately the

same as sand. Of course the compressibility of clay makes it act more like an elastic material with an infinite boundary, but at the same time, the cohesion of clay together with the plasticity tend to modify the effects of compressibility by an indeterminate amount, so further consideration here is omitted.

F. Identification of Soil Samples

The descriptions of soil samples used in this project are of minor concern since characteristics rather than numerical values of the modulus of soil reaction were investigated. Except for the moist plaster sand referred to on page 89b all soil samples used are described in Table 5. No description was available for the moist plaster sand. The clay tested in this project was a one to one mixture by weight of the clay and gumbo till described in the last two columns of Table 5. Mixing was required to provide a sample large enough for testing.

Table 5. Sources and engineering properties of soil samples

Item	Soil sample			
	White sand	Loess	Clay	Gumbo till
Iowa Exp. Sta. No. Eng.		20-2 V	207-5	416-4
Location (County in Iowa)	Black Hawk	Harrison	Allamakee	Warren
Geologic material	Silica sand	Friable loess	Clay	Plastic clay
Soil series	St. Peter Formation	Hamburg	Fayette	Shelby
Horizon		C	C	C
Sample depth		39-40 ft.	10-11 ft.	83-106 ft.
Textural percentages				
Gravel (2 mm.)		0.0	0.0	1.3 (out)
Sand (2-0.074 mm.)	Nearly 100	0.3	0.8	32.6
Silt (0.074-0.005 mm.)		82.7	76.7	29.1
Clay (0.005 mm.)		17.0	22.5	37.0
Textural classification (B.P.R.)	Sand		Silty clay loam	Clay
Atterberg limits				
Liquid limit (per cent)		33.5	29.7	38.2
Plastic limit (per cent)		27.8	22.9	15.1
Plasticity index	N.P.	5.7	6.8	23.1
Engineering classification (A.A.S.H.O.)	A-3	A-4 (8)	A-4 (8)	